

The train/embankment thought experiment, Einstein's second postulate of special relativity and relativity of simultaneity

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Abstract

The relativistic time dilatation effect and Einstein's second postulate of special relativity are used to analyse train/embankment thought experiments, both Einstein's original one, and an extension where observers on two trains moving at different speeds, as well as on the embankment, are considered. Whereas the relativistic analysis of Einstein's experiment shows, in contradiction to Einstein's interpretation, no 'relativity of simultaneity' effect, the latter is apparent for certain events in the two-train experiment. The importance of relativistic kinematics—embodied for photons in Einstein's second postulate—for the correct interpretation of the experiments is pointed out and demonstrated by detailed calculation of a related example.

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1 Introduction

Einstein's Train-Embankment Thought Experiment (TETE), introduced in his popular book 'Relativity, the Special and General Theory' [1], first published in 1920, has been used in many introductory textbooks¹ and articles in the pedagogical literature [6], to introduce the concept of 'relativity of simultaneity' of Special Relativity (SR) before considering the space-time Lorentz Transformation (LT).

Since the complete relativistic interpretation of the TETE depends both on direct physical consequences of the space-time LT —the time dilatation effect— and an understanding of the relativistic kinematics of photons as embodied in Einstein's second postulate of SR, the possibility of introducing 'relativity of simultaneity' in this simple way is illusory. This will become clear later in the present paper. However, as will be shown, a somewhat more sophisticated TETE involving observers on the embankment and in two trains, with suitably chosen speeds, does indeed demonstrate the existence of a genuine relativity of simultaneity effect for certain events. In contrast, the correct relativistic analysis of Einstein's original TETE shows that the 'lightning strokes' will be judged to be simultaneous by both the train and the embankment observers, in contradiction to Einstein's conclusion.

The fallacy in Einstein's reasoning is simple to understand. If the experiment were to be performed with non-relativistic massive particles of constant velocity replacing photons, an analysis of the various space-time events could be performed in either the embankment or the train frames and would produce identical results. In the case of photons or massive relativistic particles, as shown explicitly in the example discussed in the Appendix, this is no longer the case. Then a correct analysis of events in the train frame requires that such events be specifically considered. This was not done in Einstein's interpretation, where only events observed in the embankment frame were considered.

The structure of the paper is as follows: In the following section, after a brief discussion of the simplest axioms which may serve as the basis for SR, two consequences of SR —Invariance of Contiguity (IC) and the time dilatation (TD) effect— are derived. Application of IC, TD and Einstein's second postulate of SR are sufficient for complete analyses of the TETEs discussed in later sections of the paper. In particular, explicit use of the space-time LT is not required. Section 3 presents Einstein's original TETE and discusses it, and Einstein's interpretation of it, in a critical manner. A similar experiment where photons are replaced by sound signals, either in the atmosphere or in the interior of the train, is also discussed. In Section 4 a more elaborate TETE with two trains and a well-defined procedure for synchronous production of light signals is described and analysed. It is shown that events corresponding to light signals, triggered by coincidence detection of two other light signals in different inertial frames, show a genuine relativity of simultaneity effect quite distinct from the one proposed by Einstein. Section 5 contains a discussion of the distinction between 'relative velocity' and 'speed' as applied in the TETE as well as the closely related operational meaning of Einstein's second postulate. The latter is also shown to be a direct consequence of the relativistic kinematics of massless particles [7]. Section 6 contains a detailed discussion of a recent paper by

¹For example, Refs [2, 3]. See also numerous citations in Refs [4, 5].

Nelson [4] that gives a re-interpretation of Einstein's TETE. Nelson finally makes the same fundamental mistake, mentioned above, of attempting an analysis of the problem purely in terms of embankment frame events, although earlier in the paper came close to the correct solution of the problem. This was rejected due to a misapplication of IC to different events (falsely assumed to be the same) in the two frames. A summary and the conclusions of the present paper are to be found in Section 7. An appendix contains an analysis of the TETE from a different point of view. Instead of analysing light signals produced by the lightning strokes, the latter are replaced by pulsed laser beams in the embankment frame and the question of the simultaneity or non-simultaneity of the events where the laser pulses strike either the embankment at points aligned with the ends of the moving train, or the train itself, is considered in both the embankment and train frames. The results obtained demonstrate immediately the fallacy of Einstein's embankment-frame-only interpretation of the TETE.

Previous papers by the present author have pointed out the spurious nature of the correlated 'Length Contraction' (LC) and 'Relativity of Simultaneity' (RS) effects derived by misapplication of the space-time LT [8, 9, 10, 11]. These effects were invoked in Nelson's final 'relativistic' interpretation of the TETE. The reader is referred to these papers for a critique of the conventional interpretation of the space-time LT, and particularly to Ref [10] in which the essential arguments are concisely presented. However these arguments are also given in the present paper in Section 4 (Eqn(4.2) and Section 6 (Eqns(6.20)-(6.12)). Recognition of the spurious nature of these LT related RS and LC effects is essential to fully understand the relativistic analyses of TETEs presented in this paper.

2 Axiomatic bases of special relativity. Invariance of contiguity

Einstein's original formulation of special relativity (SR) was founded on two main postulates ²

(E1) The dynamical laws of nature are the same in any inertial frame.

(E2) The speed of light is the same in any inertial frame, and is independent of whether the source is stationary or in motion.

One aim of the present paper is to discuss the precise operational meaning of the postulate E2; however, as was realised shortly after the advent of SR [12], the space-time LT, from which all predictions of SR may be obtained, can be derived without invoking E2. See [13, 7] for surveys of the related literature. For example, in Ref [7], the LT is derived from the requirement that it is a single-valued function of the space-time coordinates and the following postulate, that is a weak operational statement of the relativity principle:

²Einstein stated explicitly only the two postulates E1 and E2 in Ref. [17], but at least three other postulates: linearity of the equations, spatial isotropy, and the reciprocity postulate (see below) were tacitly assumed.

(MRP) Reciprocal measurements of similar rulers and clocks at rest in two different inertial frames, by observers at rest in these frames, yield identical results.

It will be instructive, in the present paper, in discussing events observed from different inertial frames, to check that this Measurement Reciprocity Postulate (MRP) is respected in each case. Unlike in E1, no dynamical laws are in invoked by the MRP. Another general property of SR that will be found of great importance when comparing different interpretations of the TETE is Invariance of Contiguity (IC) which may be derived directly from the LT:

(IC) Two events which are in space time contiguity in one inertial frame are so in all inertial frames.

Pairs of space-time contiguous events may be, in an obvious way, ‘ $xt-$ ’, ‘ $xyt-$ ’, or ‘ $xyzt-$ contiguous’. In the following it will be sufficient to consider ‘ $xt-$ contiguous’ events. The proof of IC is simple. The space-time LT relating the space-time coordinates of events as specified by synchronised clocks in two inertial frames S and S’ is:

$$x' = \gamma_v(x - vt) \quad (2.1)$$

$$t' = \gamma_v(t - \beta_v x/c) \quad (2.2)$$

$$y' = y \quad (2.3)$$

$$z' = z \quad (2.4)$$

where $\beta_v \equiv v/c$, and $\gamma_v \equiv 1/\sqrt{1 - \beta_v^2}$. In (2.1)-(2.4) the frame S’ moves with speed v along the common x , x' axis of S and S’. This usual form of the LT, as derived by Einstein, is valid provided that the space time origin in S’ is coincident with the x' coordinate of the transformed event. The appropriate LT when this condition is not satisfied is discussed in Section 6 below. Any two events in S: (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) and the corresponding events in S’: (x'_1, y'_1, z'_1, t'_1) and (x'_2, y'_2, z'_2, t'_2) that satisfy (2.1)-(2.4) also respect the interval relations, that are independent of the choice of the origins of coordinate systems in S and S’:

$$x'_1 - x'_2 = \gamma_v[x_1 - x_2 - v(t_1 - t_2)] \quad (2.5)$$

$$t'_1 - t'_2 = \gamma_v[t_1 - t_2 - \beta_v(x_1 - x_2)/c] \quad (2.6)$$

$$y'_1 - y'_2 = y_1 - y_2 \quad (2.7)$$

$$z'_1 - z'_2 = z_1 - z_2 \quad (2.8)$$

If events 1 and 2 are ‘ $xt-$ contiguous’ in S. i.e. if $x_1 = x_2$ and $t_1 = t_2$ It follows from (2.5) and (2.6) that also $x'_1 = x'_2$ and $t'_1 = t'_2$. Thus the events are contiguous in S’ for any value of v , that is, in any inertial frame. An immediate and important corollary for the discussion of the TETE is that if the x - t world lines of three physical objects intersect at a common point in any inertial frame, i.e. if the three events at the intersection are $xt-$ contiguous, they must intersect at a common point in any inertial frame.

This demonstration is the only direct use of the space-time LT in following discussion of TETEs. It will be found possible to completely analyse the different thought experiments considered by noting that a light-transit time in a given inertial frame between objects at

rest in that frame, constitutes a ‘photon clock’ [14] that measures the proper time in that frame. As do all clocks when in uniform motion, such clocks display the time dilatation (TD) effect. The prediction of the TD effect relating a time interval, $\Delta t'$, recorded by a clock at rest in the frame S' , but as observed from the frame S , to corresponding time interval, $\Delta\tau$, as measured by a clock at rest in S , may be derived from the time-like invariant interval relation:

$$c^2(\Delta\tau)^2 - (\Delta x)^2 = c^2(\Delta t')^2 - (\Delta x')^2 \quad (2.9)$$

A clock at a fixed position in S' has $\Delta x' = 0$. Since also $\Delta x = v\Delta\tau$ it follows from (2.9) that:

$$c^2(\Delta\tau)^2(1 - \frac{v^2}{c^2}) = c^2\frac{(\Delta\tau)^2}{\gamma_v^2} = c^2(\Delta t')^2 \quad (2.10)$$

or

$$\Delta\tau = \gamma_v\Delta t' \quad (2.11)$$

Consideration of a clock at rest in S , but observed from S' , leads to the similar formula:

$$\Delta\tau' = \gamma_v\Delta t \quad (2.12)$$

where τ' is the time recorded by a clock at rest in S' , and Δt is the corresponding time recorded by a similar clock at rest in S but observed from S' .

It is very important to note, in the analysis of space-time problems in special relativity, the existence of the four different time intervals $\Delta\tau$, $\Delta\tau'$, Δt and $\Delta t'$. Where τ and τ' are observed proper times and t and t' are the apparent times of moving clocks. The intervals $\Delta\tau$ and $\Delta t'$ relate to one possible experiment and $\Delta\tau'$ and Δt to a different, although reciprocal, in the sense of the MRP, one. Thus when the LT are used to analyse experiments it always gives a relation between a proper time and an apparent time, never between two proper or two apparent times. The erroneous identification of τ with t or of τ' with t' has resulted in many incorrect predictions of relativistic effects, particularly in classical electrodynamics [15]. Another example of this error of interpretation of the LT is provided by Nelson’s analysis of the TETE [4] to be discussed below in Section 6. Indeed, the logical absurdity of setting $\Delta\tau = \Delta t$ and $\Delta\tau' = \Delta t'$ thus mixing up the time intervals recorded in different experiments becomes evident on substituting these relations into (2.11) and (2.12) to yield the equations:

$$\Delta\tau = \gamma_v\Delta\tau' \quad (2.13)$$

$$\Delta\tau' = \gamma_v\Delta\tau \quad (2.14)$$

Substituting (2.14) into (2.13) gives $\gamma_v^2 = 1$ or $v = 0$, whereas γ_v in (2.11) and (2.12) can take any positive real value. On the other hand, the substitutions $\Delta t \leftrightarrow \Delta t'$ and $\Delta\tau \leftrightarrow \Delta\tau'$ correspond to the valid operation of replacing a time dilatation experiment by a reciprocal one, which has the effect of exchanging Eqns(2.11) and (2.12).

The convention of using the symbol τ to denote the observed time of a clock at rest and t to denote the observed time of a clock in uniform motion is followed throughout the present paper.

Eqns(2.11) and (2.12) may also be written in a more explicit notation as:

$$\Delta\tau_{rest} = \gamma_v\Delta\tau'_{move} \quad (2.15)$$

$$\Delta\tau'_{rest} = \gamma_v \Delta\tau_{move} \quad (2.16)$$

where all intervals refer to proper times and the subscripts specify whether the corresponding clock is in motion or at rest. It is this form of the TD interval relation that is used to extract the proper lifetime of an unstable particle $\Delta\tau'_{move}$ from the measurement of the corresponding laboratory lifetime $\Delta\tau_{rest}$ knowing the value of the relativistic parameter γ_v from kinematics or direct velocity measurement.

3 Einstein's presentation of the TETE

In his original presentation of the TETE [1], Einstein considered a train moving with uniform velocity, v , along an embankment. In the following, the inertial frame of the embankment is denoted by S , that of a train by S' or S'' . The positive x -axis is parallel to the direction of motion of the train. Two lightning strokes hit points A and B of the embankment, simultaneously in S . The spatially contiguous points on the train, at the instant of the lightning strokes are denoted by A' and B' . It is asked whether the lightning strokes will be perceived by an observer on the train as simultaneous, or not simultaneous. To answer this question, observers are introduced at M and M' on the embankment and the train respectively. M is midway between A and B and M' is midway between A' and B' . The production process of the light signals in the frames S and S' associated with the lightning strokes and observed at M and M' was not discussed by Einstein. As will be seen, this is not important for Einstein's interpretation since no observations performed in the frame S' are considered.

Einstein's analysis of the TETE is presented in Fig.1. For definiteness, the 'observers' at M and M' are replaced by photon detectors D and D' . Using his light signal synchronisation procedure, Einstein argues that the events at A and B (or at A' and B') will be simultaneous provided that the corresponding light signals, as observed in the embankment frame, are simultaneous at D (or at D'). From this, it is concluded that the photon (or light signal) initiating events are simultaneous for the observer on the embankment, but not for the observer on the train. Clearly (assuming all photons have the same speed in a given inertial frame) the events will be judged to be simultaneous by the observer on the embankment (see Fig.1c). Einstein gives the following argument concerning the observation of the light signals on the train [16]:

Now in reality (considered with reference to the railway embankment) he (i.e. the observer in the train) is hastening towards the beam of light coming from B , while he is riding on ahead of the beam of light coming from A . Hence the observer will see the beam of light emitted by B earlier than that emitted from A . Observers who take the railway train as the reference-body must therefore come to the conclusion that the lightning flash at B took place earlier than the lightning flash at A . We thus arrive at the important result: Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train and vice versa (relativity of simultaneity).

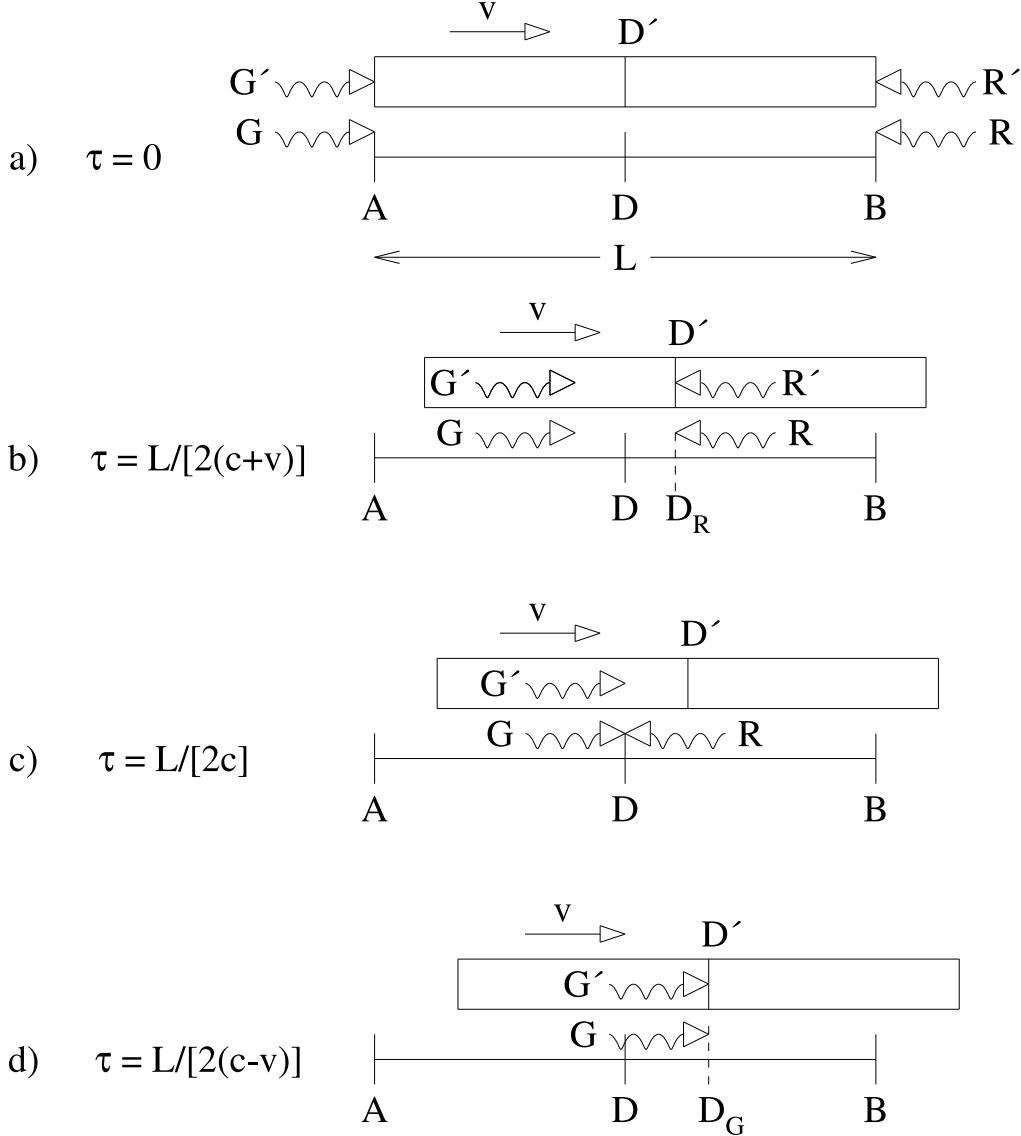


Figure 1: *Einstein's interpretation of the TETE in the embankment frame.* a) $\tau = 0$, photons G', G , R' and R are emitted from lightning stroke events at A and B . b) $\tau = L/[2(c + v)]$, photons, R , R' are spatially contiguous with D_R and D' respectively. c) $\tau = L/[2c]$ photons G and R are detected in coincidence by D . d) $\tau = L/[2(c - v)]$, photons G , G' are spatially contiguous with D_G and D' respectively. In this figure $v = c/4$. The photons G' and R' that are detected by D' in the train are assumed to be xt -contiguous with the photons G and R , respectively, that are detected by D on the embankment. Since the analysis is performed entirely in the embankment frame the events $R'-D'$; and $G'-D'$ considered by Einstein are actually the embankment frame events $R-D_R$ and $G-D_G$, not events in the train frame. Thus the essential problem is not addressed. In order to discuss the simultaneity (or lack of it) of events in the train frame, an analysis of such events, in the train frame itself, must be performed. As discussed in the Appendix, if the photons are replaced by non-relativistic massive particles analyses in the train and embankment frames are equivalent, so that a correct space-time analysis may be performed in either frame. This is no longer the case in special relativity, invalidating Einstein's interpretation.

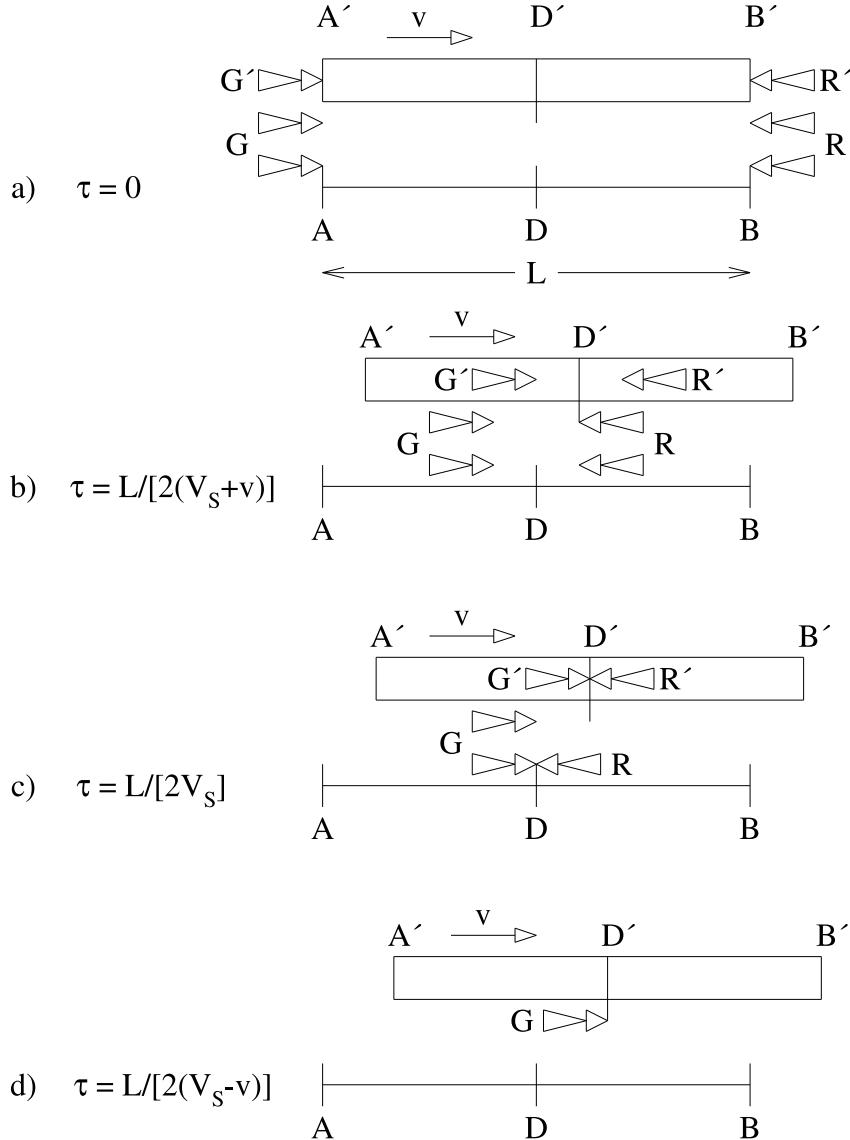


Figure 2: *The Einstein TETE with light signals replaced by sound signals. If uniform sound propagation, in the stationary atmosphere, unperturbed by the motion of the train, is considered (signals G, R) results strictly analogous to Einstein's for the case of light signals are obtained with the replacement $c \rightarrow V_S$. If the sound signals in the train instead move in air that is at rest relative to the train, (signals G', R'), the signals are observed simultaneously in the train [c]). In this figure $v = V_S/4$. In view of Einstein's second postulate, replacing the speed V_S of sound by the speed of light c , it may be expected that the light signals would also be observed simultaneously in the train, in contradiction to Einstein's conclusion.*

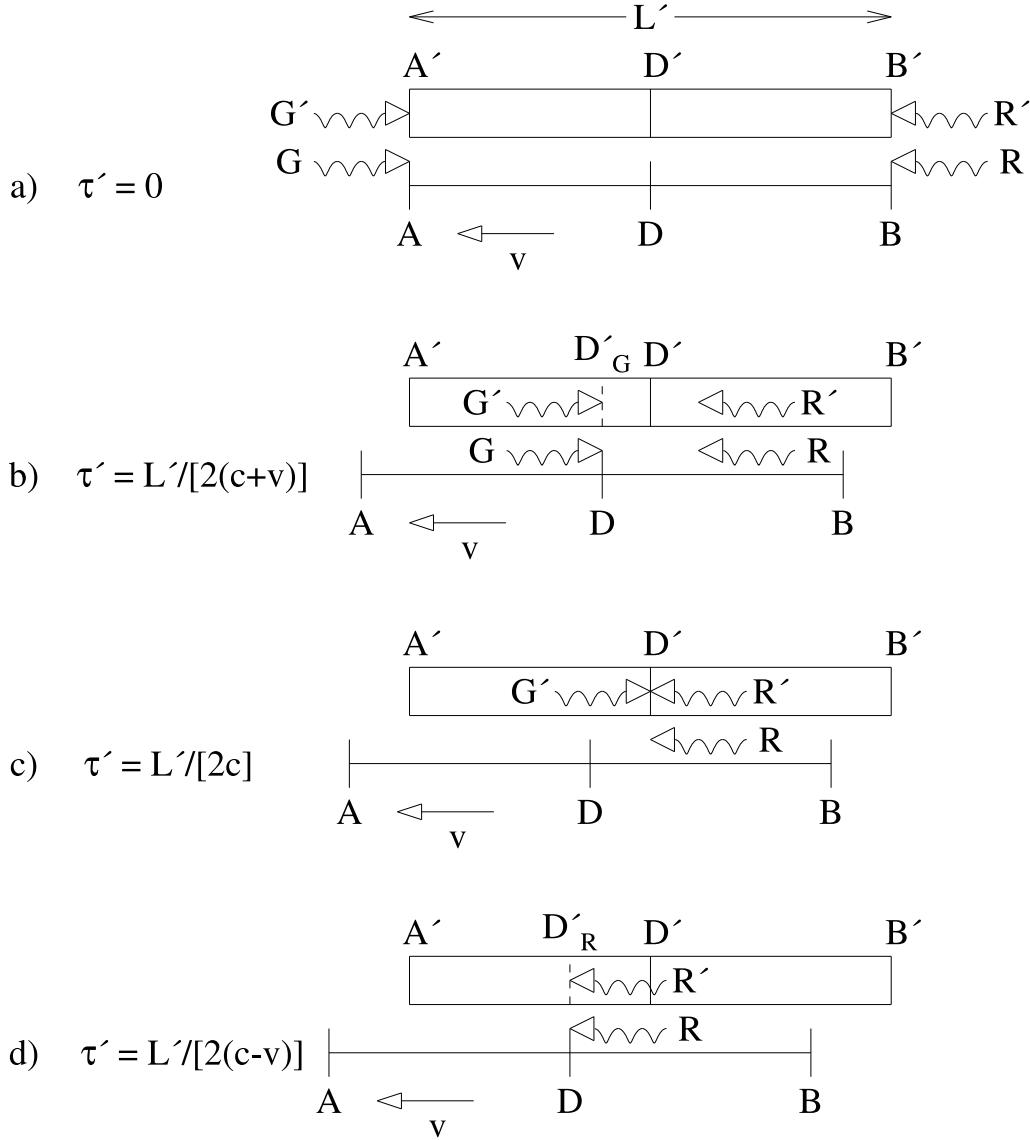


Figure 3: *Einstein's interpretation of the TETE in the train frame. As in Fig.1, $v = c/4$. It is assumed that the lightning stroke events are also simultaneous in this frame. As shown in the text, this is the case in both Galilean and special relativity. The events $G'-D'_G$, $G'-R'-D'$ and $R'-D'_R$ are specified in the train frame. The times of the events are obtained from those shown in Fig.1 by exchange of primed and unprimed quantities, so that the experiments shown in this figure and Fig.1 are ‘reciprocal’ ones. In Einstein’s interpretation the events $G'-D'_G$ and $R'-D'_R$ in the train frame are wrongly identified with putative embankment frame events $G-D$ and $R-D$ respectively.*

Actually, however, all the observations described in this passage are performed by observers in the embankment frame, never by an observer on the train. When the photon R' arrives at D' (Fig.1b) before the photon G' arrives at D' (Fig.1d) what are observed are events in S , not in S' . To specify precisely, in an operational manner, these events which are observed in S , it is convenient to introduce the detectors D_R and D_G , at rest in S , that are spatially contiguous with D' at the apparent instants of photon coincidences with the latter detector (see Fig.1b and 1d). Since these clocks are at rest in S , a definite time (the proper time in S) can be assigned to the R - D_R and G - D_G coincidence events. This is not the case for the time in S' corresponding to apparent R' - D' and G' - D' coincidences. Since the detector D' is in motion, the time of the photon detection events in S' , which is what is essential for problem under consideration, cannot be *a priori* identified with the S -frame times of the events R - D_R and G - D_G , which is what is assumed in Einstein's interpretation. So, contrary to Einstein's statement, no conclusion can be drawn concerning the simultaneity, or non-simultaneity of the observation of the photons R' and G' in the frame S' . Indeed, the postulate E2, stating the equality of the speed of light in the frames S and S' , is not invoked in Einstein's argument, as it must be (and is in the following section) in order to obtain the correct prediction of SR concerning the simultaneity (or lack of it) of the photon detection events observed in S or S' . For this it is necessary to consider events not only in the frame of the embankment, as Einstein does, but also events in the frame of the train.

The argument given in Einstein's passage quoted above is very similar to that presented in the original paper on SR [17] where 'relativity of simultaneity' was first introduced. A 'photon clock' in motion was compared with one at rest, but only events in a single inertial reference frame were discussed. Indeed, at this point in Ref. [17], no meaningful discussion of relativity of simultaneity was possible because the LT had yet to be derived. Similarly in Ref. [1], the LT necessary for a correct analysis of relativity of simultaneity is introduced only in Chapter XI after previously introducing the TETE in Chapter IX and relativity of distance (also based on the TETE) in Chapter X.

In fact, the sequence of events seen in a single inertial frame shown in Fig.1 would be identical in SR, where postulate E2 applies, or in the preferred frame (aether rest frame) of a theory where E2 does not apply, but instead light may have different speeds in different inertial frames, in the case that the embankment is associated with this preferred frame. Indeed, as shown in Fig.2, the pattern of events is the same if the photons in Fig.1 are replaced by sound signals in air, if the train does not drag the air in the regions where the sound signals are propagating. If however the sound signals were detected *inside* the train (assuming that the centre of mass of the air in the train is rest in S') the production events would be judged simultaneous by both the train and embankment observers (Fig.2c). However, the sound signals in S would not be seen by an observer in this frame to arrive simultaneously at the *position of D' in S* (Fig.2b and 2d). In the limit where the speed of sound in air, V_S , is much less than the speed of light, c , not only will the R and G (R' and G') signals be received simultaneously by the detectors D (D') but also the G' - R - D and G - D - R coincidence events will be seen to be simultaneous by observers in both S and S' (Fig.2c). As will be seen in the following section, this is no longer the case when corrections of $O((V_S/c)^2)$ are included.

Since the postulate E2 states that the speed of light signals is the same in S and S' ,

regardless of the motion of the source, it would seem that the situation shown in Fig.2 for the motion of the sound signals in the train should be strictly analogous to that of the light signals in vacuum inside the train, replacing V_S by c . The physics underlying the propagation of the signals is quite different in the two cases (constant speed of sound relative to air in one case, Einstein's postulate E2 in the other) but, in both cases, the signals would be expected, contrary to Einstein's conclusion, to be simultaneous, both in the train and on the embankment.

Einstein chose to analyse the problem entirely in the reference frame of the embankment observer. What happens if it is instead analysed in the reference frame of the observer on the train? This is shown in Fig.3, where it is assumed that the lightning stroke events are also simultaneous as seen by local observers³ in the train frame. The validity of this assumption is further discussed below, as well as in the Appendix. Performing exactly the same analysis as Einstein, but instead considering events only in the frame S' instead of only in the frame S , it would be concluded, from Einstein's argument, that the production events of the photons G' and R' are simultaneous (Fig.3c) whereas those of G and R are not (Fig.3b and Fig.3d). This different behaviour is perfectly compatible with the relativity principle as embodied in the MRP. However, there is an apparent violation of invariance of contiguity if it is assumed that the G - R - D coincidence in Fig.1 and the G - D and R - D coincidences in Fig.3 represent the same event. The x - t world lines of G , R and D intersect in a point in S (Fig 1c) but not in S' (Fig.3b and Fig.3d) Similarly the x' - t' world lines of G' , R' and D' intersect in a point in S' (Fig.3c), but not in S (Fig.1b and Fig.1d). Thus it appears that Einstein's claimed 'relativity of simultaneity' effect is incompatible with IC, that is, as shown in the previous section, a necessary consequence of the LT. However, as discussed above, such an identification of all photon detection events as common events, viewed either in S or in S' , is fallacious. The non-simultaneous events in S are R - D_R and G - D_G not R' - D' and G' - D' respectively, while the observed events in S' are (see Fig.3) R' - D'_R and G' - D'_G not R - D and G - D in S . Indeed, R - D_R , G - D_G , R - G - D , R' - D'_R , G' - D'_G and R' - G - D' are all physically distinct events that may be viewed either from either the train or the embankment frames. There is therefore no conflict with Invariance of Contiguity. This point is further discussed in Section 6 below. In Einstein's interpretation it is assumed that only the events R - D (actually R - D_R), G - D (actually G - D_G) and R - G - D exist, and these events are considered only in the embankment frame.

The following section presents an analysis of a more elaborate TETE that will be found to respect IC and which does indeed lead to the prediction of 'relativity of simultaneity' of certain events observed in different frames of reference, as a prediction of SR, but in a way quite different to that suggested by Einstein in the passage quoted above, or in the original SR paper. This analysis, in conjunction with the above critique of Einstein's interpretation, will also shed light on the correct operational meaning of the postulate E2, to be discussed in more detail in Section 5 below.

³i.e., observers situated at A' and B' .

4 The TETE as a photon clock: Two trains for relativity of simultaneity

A version of the TETE approximating more closely an actual experiment will now be considered. A schematic is shown in Fig.4. There are two trains T_U and T_L moving with velocities v and $w = c^2(\gamma_v - 1)/(v\gamma_v)$ respectively in the positive x -direction relative to the embankment E . The latter velocity is chosen such that E and T_U have equal and opposite velocities, with absolute value w , relative to T_L . The rest frames of T_U , T_L and E are denoted by S' , S'' and S respectively. Both trains and the embankment are equipped with light sources G' , R' , Y' ; G'' , R'' , Y'' ; G_U , R_U , Y_U ; G_L , R_L , Y_L . The light sources G' , R' ; G'' , R'' ; G_U , R_U ; G_L , R_L are connected to an electronic triggering system and sensors which fire the sources when G' , G'' , G_U and G_L are aligned with the point A on the embankment and R' , R'' , R_U and R_L are aligned with the point B on the embankment. The length of the train is chosen so that all eight sources fire simultaneously in the frame S . This instant, which is the one corresponding to the configuration shown in Fig.4, defines the origin of the time coordinates in all three frames of reference: $\tau = \tau' = \tau'' = 0$. The photon detectors D' , D'' , D_U and D_L are situated mid-way between the corresponding G and R photon sources, and arranged in such a way that D' , D'' , D_U , D_L detect only the photons emitted by the sources G_U and R_U , G_L and R_L , G' and R' , G'' and R'' , respectively. All of the detectors are double-sided and equipped with coincidence electronics such that, in the case that the photons are recorded, simultaneously, on both sides of the detector, the light sources Y' , Y'' , Y_U and Y_L will fire. The purpose of these sources is to enable the observation of coincidence events in one frame in both of the other frames. It will be found that it is precisely these coincidence events (not the G and R photon production events, that are simultaneous in all frames) that will show a ‘relativity of simultaneity’ effect. As in the previous section, the labels G , R , Y are used indiscriminately to indicate photons, or their sources. In the present TETE they can also be conveniently identified with the colours Green, Red and Yellow of the respective photons. Thus the Yellow signal indicates a space-time coincidence (xt -contiguity) of Green and Red photons. Suitable wavelength filters in front of the detectors can ensure that only appropriately coloured photons are detected.

The TETE shown in Fig.4 will now be analysed by applying Einstein’s postulate E2 of SR in each of the frames S , S' and S'' . In Fig.5 is shown the sequence of events in the train T_U (frame S') and on the embankment (frame S) as observed from the embankment, while Fig.6 shows the sequence of events in the train T_U and on the embankment as observed from the train T_U . For example, at $\tau = 0$ (Fig.5a), the photons emitted from G' and R' start their transit towards D_U . Both of these sources are moving with velocity v in the +ve x -direction relative to D_U . Similarly, in S' (Fig.6a) the sources R_U and G_U move in the -ve x' -direction relative to D' and the corresponding photons are emitted in the direction of D' . Application of the postulate E2 in either frame predicts the observation of the Yellow signals shown in Fig.5b, 5c, 6b and 6c. At time $\tau = L/(2c)$ the photons G' and R' arrive in coincidence at D_U and the source Y_U fires. The distance L' between the sources G' and R' (compare with Fig.3a) can be found by use of the space-like invariant interval Δs between arbitrary events at A' and B' expressed in terms of space-time coordinate

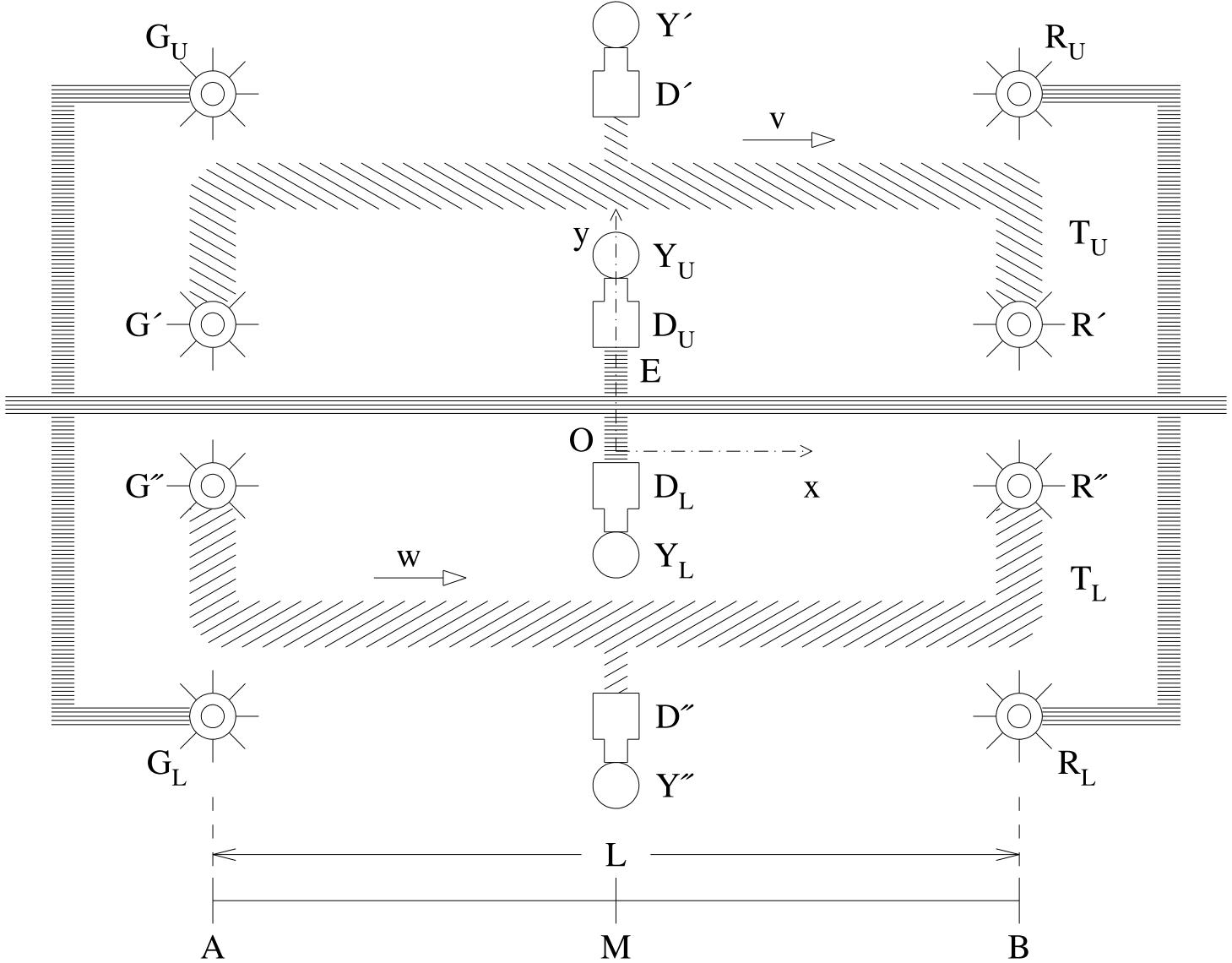


Figure 4: A TETE with two trains T_U and T_L moving to the right with velocities v and $w = c^2(\gamma_v - 1)/(v\gamma_v)$ respectively. The velocity w is chosen so that, in the rest frame of T_L , T_U and the embankment E have equal and opposite velocities of absolute value w . Both trains and the embankment are equipped with light sources G and R and photon detectors D (see text for details). Each detector is also equipped with a light source Y that fires in the case that right- and left-moving photons are detected in coincidence. The G and R light sources fire when they are all aligned with the points A and B on the embankment, as shown in the figure.

intervals in the frames S and S':

$$(\Delta s)^2 \equiv (\Delta x)^2 - c^2(\Delta \tau)^2 = (\Delta x')^2 - c^2(\Delta t')^2 \quad (4.1)$$

In Fig.5a, $\tau = t' = \Delta \tau = \Delta t' = 0$, so from (4.1),

$$\Delta s = \Delta x \equiv x_{B'} - x_{A'} = L = \Delta x' \equiv x'_{B'} - x'_{A'} \equiv L' \quad (4.2)$$

This equation shows that the spatial separation of two physical objects at rest in an inertial frame is a Lorentz-invariant quantity [18] that takes the same value when measured in any inertial frame. There is no 'relativistic length contraction' effect if the proper frame of the objects is in motion relative to another inertial frame [8, 9, 10, 11]. By considering events at A'' and B'' on the train T_L it is shown in a similar manner that $x''_{B''} - x''_{A''} \equiv L'' = L$. It follows that G_U and G_R will arrive at D' in S' at the time $\tau' = L/(2c)$. Allowing for the TD effect, Eqn(2.11), (T_U constitutes a moving photon clock) the observer in S will see the coincidence of G_U and R_U at D' (i.e. the signal Y') at time $\tau = \gamma_v L/(2c)$ (Fig.5c). Thus, contrary to the conclusions of Einstein's analysis of a similar TETE, observers in both S and S' will judge the photon production events to be simultaneous. However, unlike for the case of sound signals, where terms of $O(V_S/c)^2$ and higher are neglected, shown in Fig.2c, the $G_U - R_U - D'$ coincidence is observed at a later time than the $G' - R' - D_U$ one. Note however, that the transit of the sound signals in the train in Fig.2 constitutes a clock just as well as the photon transits in Fig.5, so that if terms of $O(V_S/c)^2$ and higher are included the $G' - R'$ -sound signal coincidence in Fig.2c is also predicted to occur later than the $G - R$ coincidence in this figure.

Fig.6 shows the same sequence of events as in Fig.5, but as seen by an observer at rest in the train T_U . The $G_U - R_U - D'$ and $G' - R' - D_U$ coincidences are observed as before, but now the former of these events is observed earlier, not later as in Fig.5. The sequences of events in Figs. 5 and 6 are an example of an application of the MRP. One is obtained from the other by simple exchange of primed and unprimed quantities. Also, unlike in Figs.1 and 3, as interpreted by Einstein, IC is respected. Observation of the signals Y' and Y_U by observers on both S and S' shows that both of them agree that the R and G photons are detected simultaneously, from which the simultaneity of the photon production events in both frames can be inferred.

In Fig.7 is shown the sequence of events seen by an observer in the train T_L that moves with velocity w along the +ve x -axis relative to the embankment and velocity w along the -ve x -axis relative to the the train T_U . In Fig.7a the configuration of the sources and detectors in the frame S'' at the time $\tau'' = 0$ is shown, together with their velocities relative to the detector D'' . Using the photon transit times in the various frames as clock signals, and taking into account the TD effect (2.11) predicts the sequence of events shown in Fig.7b and 7c. The $G_L - R_L - D''$ coincidence (signal Y'') is observed at time $\tau'' = L/(2c)$. and the $G_U - R_U - D'$ coincidence (signal Y') and the $G'' - R'' - D_L$ coincidence (signal Y_L) are observed simultaneously at the later time $\tau'' = \gamma_w L/(2c)$. Since it is clear, from the geometry of Fig.4, that the signals Y_L and Y_U are xt contiguous in S and therefore, by IC, simultaneous in all frames, it follows that the signal events Y' and Y_U or Y_L display relativity of simultaneity:

$$\text{Observed from } E \text{ (frame } S, \text{ Fig.5)} \quad \tau(Y') > \tau(Y_U) = \tau(Y_L)$$

$$\text{Observed from } T_L \text{ (frame } S'', \text{ Fig.7)} \quad \tau''(Y') = \tau''(Y_L) = \tau''(Y_U)$$

$$\text{Observed from } T_U \text{ (frame } S', \text{ Fig.6)} \quad \tau'(Y') < \tau'(Y_U) = \tau'(Y_L)$$

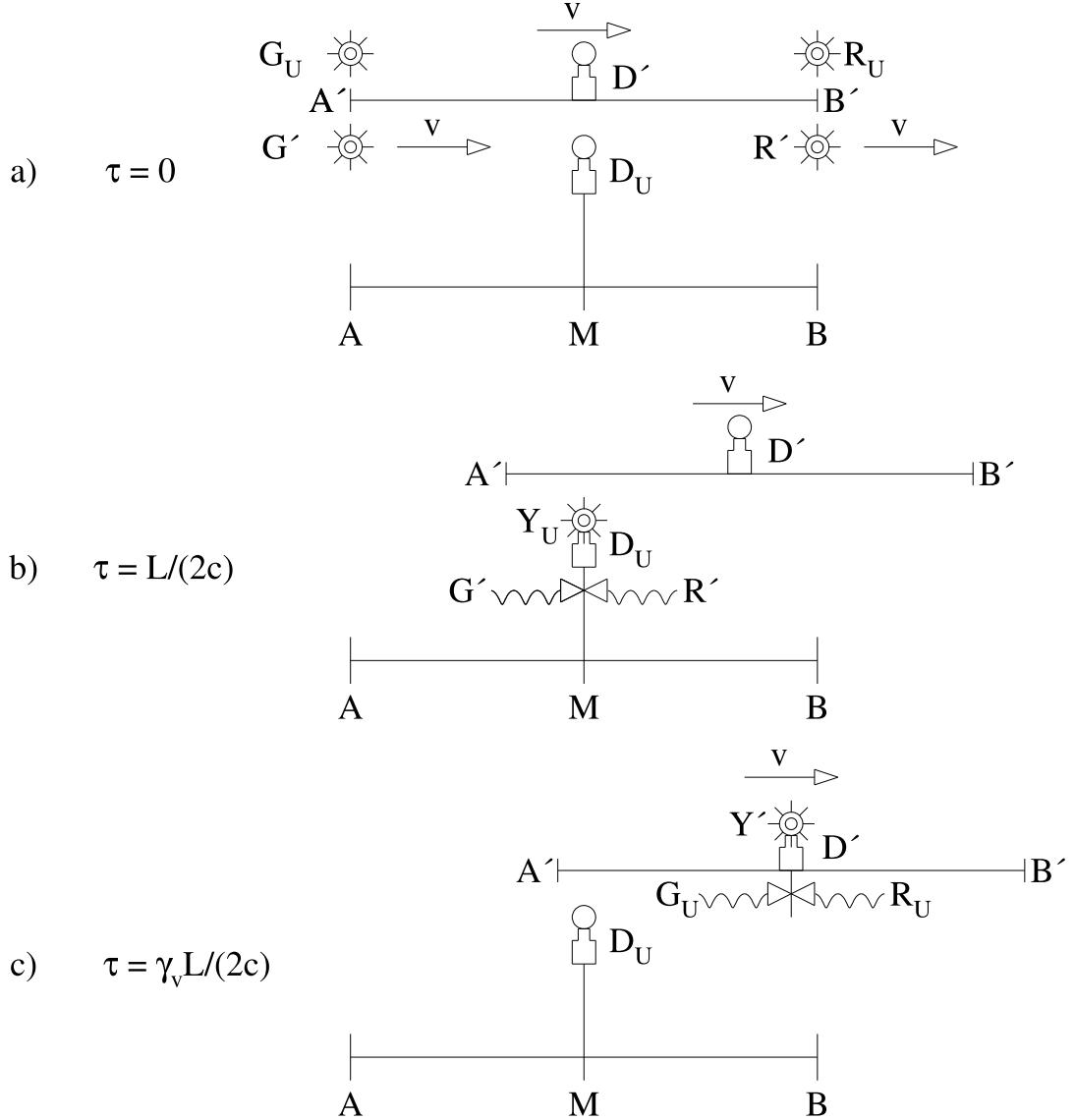


Figure 5: Events on the embankment E (frame S) and in the train T_U (frame S') as observed from the embankment. a) Sources $(G', G_U), (R', R_U)$ fire at $\tau = \tau' = 0$ when they are both aligned with the points $(A), (B)$ on the embankment. b) $\tau = L/(2c)$, photons from G' and R' are detected, in coincidence, by D_U ; Y_U fires. c) $\tau = \gamma_v L/(2c)$, photons from G_U and R_U are detected in coincidence by D' in S' ; Y' fires and is observed from S . In this figure $\beta_v = 2/3$, $\gamma_v = 3/\sqrt{5}$. For clarity, the vertical positions of the detectors are shown displaced.

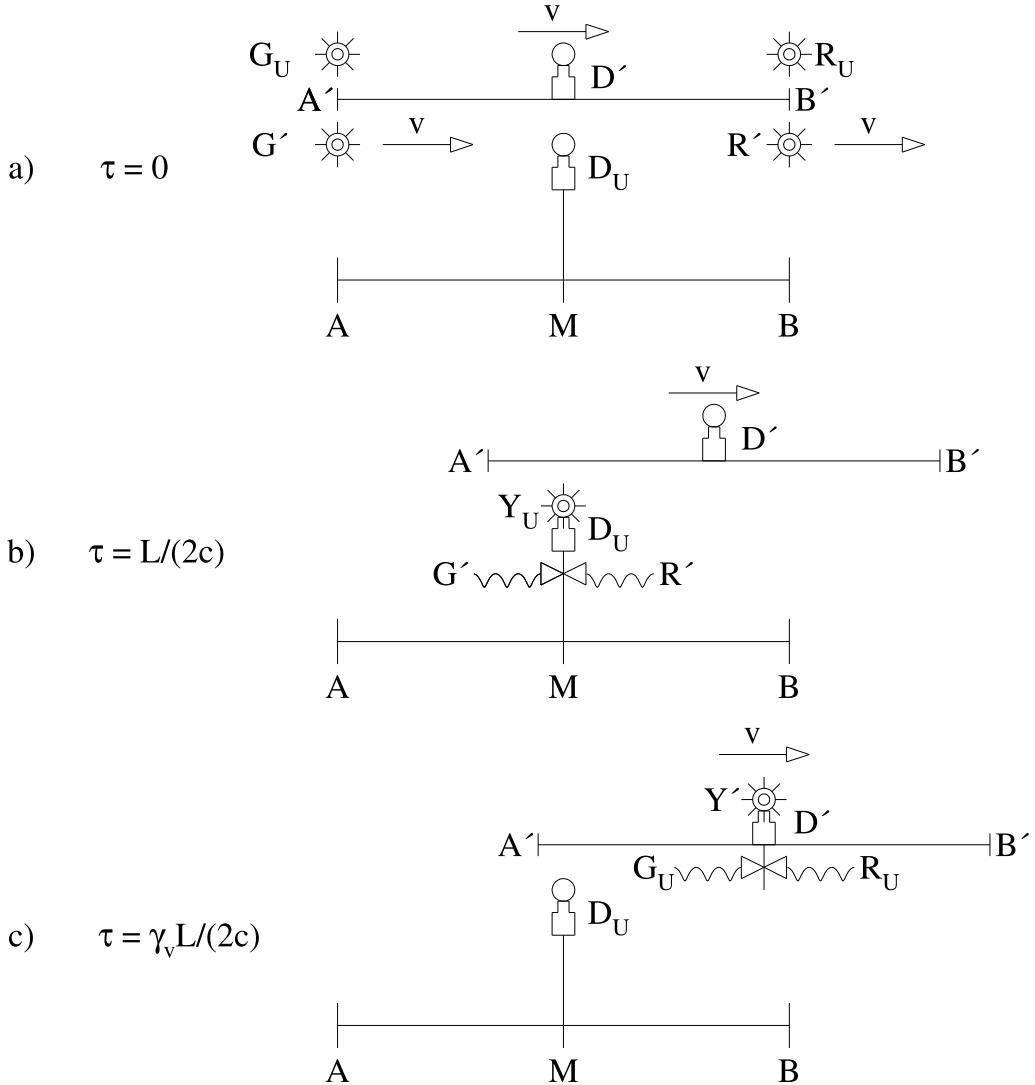


Figure 6: Events on the embankment E (frame S) and in the train T_U (frame S') as observed from the train T_U . a) Sources G' , G_U , R' and R_U fire at $\tau = \tau' = 0$. b) $\tau' = L/(2c)$, photons from G_U and R_U are detected, in coincidence, by D' ; Y' fires. c) $\tau' = \gamma_v L/(2c)$, photons from G' and R' are detected in coincidence by D_U in S ; Y_U fires and is observed from S' . In this figure $\beta_v = 2/3$, $\gamma_v = 3/\sqrt{5}$. For clarity, the vertical positions of the detectors are shown displaced.

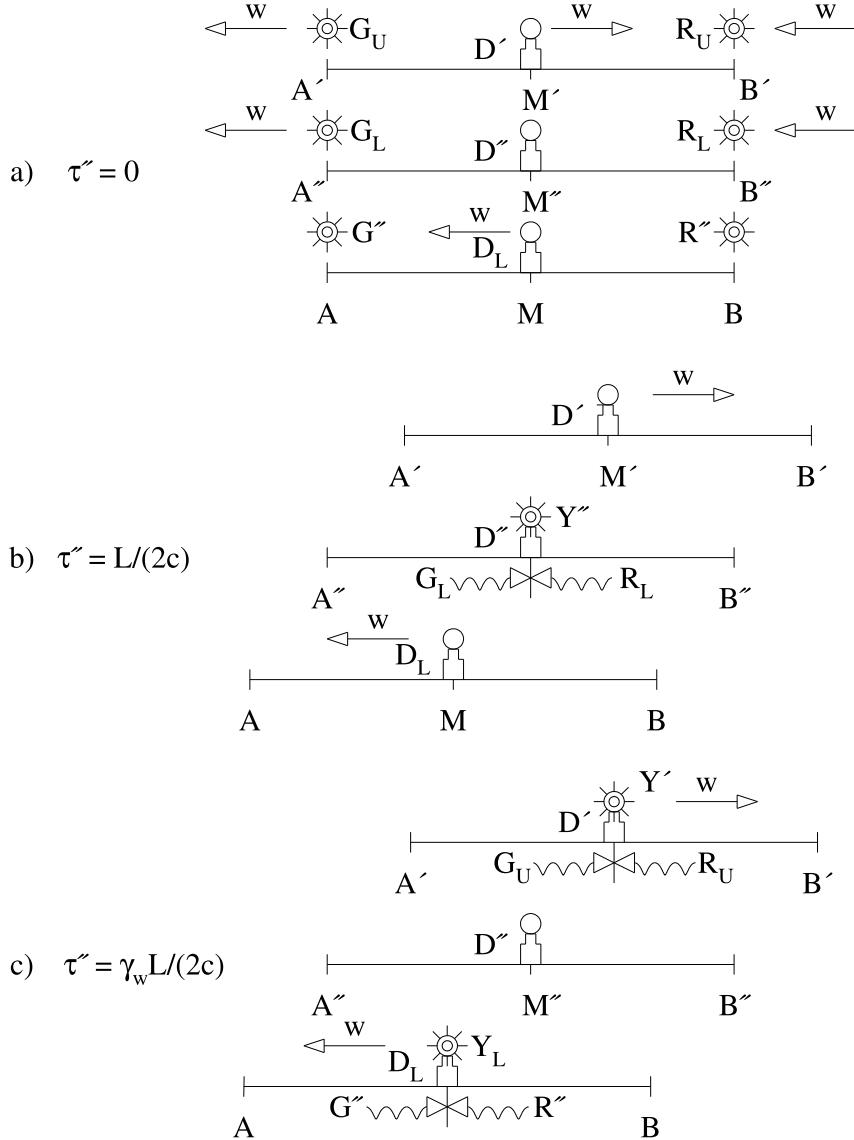


Figure 7: Events on the embankment E (frame S), in the train T_U (frame S') and in the train T_L (frame S'') observed from the train T_L . a) Sources G_U , G_L , G'' , R_U , R_L and R'' fire at $\tau = \tau' = \tau'' = 0$. b) $\tau'' = L/(2c)$, photons from G_L and R_L are detected, in coincidence, by D'' ; Y'' fires. c) $\tau'' = \gamma_w L/(2c)$, photons from G_U and R_U are detected in coincidence, by D' ; Y' fires. Also photons from G'' and R'' are detected, in coincidence, by D_L ; Y_L fires. Y' and Y_L are observed simultaneously in S'' . In this figure $\beta_v = 2/3$, $\beta_w = 0.382$, $\gamma_w = 1.082$. For clarity, the vertical positions of the detectors are shown displaced.

Since the times of the signals Y' , Y_U , Y_L and Y'' are found to be positive in all frames, and so later than the production time $\tau = \tau' = \tau'' = 0$ of the G and R photons, causality is respected in all observations i.e. no observation of an apparent emission time that is later than an apparent detection time.

The above behaviour, showing relativity of simultaneity, derived from the synchronised ‘photon clocks’ in S , S' and S'' , will of course be displayed by any other, similarly synchronised, clocks in these frames. For example, clocks, of any construction, situated at the same positions as D' , D_U , D_L and D'' and set to $\tau = \tau' = \tau'' = 0$ when they are all in xt -contiguity, as shown in the configuration of Fig.4.

5 Relative velocity and speed: The meaning of the second postulate

The complete statement of the postulate E2 given in Ref. [17] is:

Any ray of light moves in the ‘‘stationary’’ system of coordinates with the determined velocity, c , whether the ray be emitted by a stationary or a moving body. Hence:

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}} \quad (5.1)$$

where ‘time interval’ is to be taken in the sense of the definition in Paragraph 1⁴.

This definition must be treated with some care and circumspection. In particular the *relative velocity* of two photons or of a photon and some massive physical object, as observed in a particular frame of reference, may well be less than, or greater than, c .

The definition of the mean relative velocity of two objects A and B, over some interval of time $\Delta t = t_2 - t_1$, as observed in some frame of reference, is:

$$\begin{aligned} v_{BA} &\equiv \text{velocity of B relative to A} \\ &\equiv \frac{x_B(t_2) - x_A(t_2) - [x_B(t_1) - x_A(t_1)]}{t_2 - t_1} \\ &\equiv \frac{\Delta x_{BA}(t_2) - \Delta x_{BA}(t_1)}{\Delta t} \end{aligned} \quad (5.2)$$

where 1-dimensional motion is considered. For the case of three objects, each in uniform motion along a straight line, the relation $\Delta x_{AB} = -\Delta x_{BA}$, following from the definition of Δx_{AB} , gives the identity relating three relative velocities:

$$v_{BA} + v_{AC} + v_{CA} = 0 \quad (5.3)$$

Consider the velocities of the photons R’ and G’ relative to the detector D’ as viewed in the embankment frame S (Fig.1a). The geometry of the configurations shown in Fig.1a

⁴That is, as defined by appropriate clocks synchronised by the light signal procedure defined in Paragraph 1 of Ref. [17].

and Fig.1b give:

$$v_{G'D'} = c - v \quad (5.4)$$

$$v_{D'R'} = c + v \quad (5.5)$$

$$v_{R'G'} = -2c \quad (5.6)$$

The photons R' and G' certainly follow ‘light paths’ in propagating from their points of production on Fig.1a to their points of detection in Fig.1b and Fig.1d respectively, however, in neither case is the appropriate velocity in Eqn(5.1) the constant c .

The relative velocities in Eqns(5.2)-(5.6) —that between two objects in the same reference frame— must be distinguished from those appearing in the parallel velocity addition formula, which connects instead the velocities of a single physical object in different frames of reference. In each frame the velocity of the object as defined as that relative to the fixed coordinate axes of the frame. It is this velocity, and only this velocity, for the case of light, that is correctly given, in any inertial frame, by Einstein’s formula (5.1). The speed of light, the magnitude of its velocity defined in this way, is then the same in all inertial frames. It is operationally defined as the distance between the fixed points of production and detection of a photon in an inertial frame, divided by the time-of-flight of the photon as measured by synchronous clocks at rest in the same reference frame.

It is interesting to note that Einstein did not explicitly state that the speed of light measured in this way is the same in *all* inertial frames, when defining, in the passage quoted above, the postulate E2. This was done later in the paper during the derivation of the LT. Since Einstein regarded the constancy of the speed of light as a ‘law of nature’, it was, for him, already implicit in the first postulate E1.

In fact the postulate E2 is most easily understood not as a separate ‘law of nature’, under which aspect it seems highly counter-intuitive⁵, but rather as a simple kinematical consequence of the fact that light consists of massless particles. In the derivation of the LT in Ref. [7] in which electrodynamics or the propagation of light play no part, it is found that the relative velocity of two inertial frames, A and B, defined according to the formula (5.2) with either $x_A = \text{constant}$, or $x_B = \text{constant}$, has a maximum possible value, V . If one frame is the proper frame of an object of Newtonian mass, m , moving with velocity v relative to the other, the relativistic momentum, p , and energy, E , of the object in the second frame are defined by the relations:

$$p \equiv \tilde{\gamma}_v mv \quad (5.7)$$

$$E \equiv \tilde{\gamma}_v mV^2 \quad (5.8)$$

where

$$\tilde{\gamma}_v \equiv \frac{1}{\sqrt{1 - (v/V)^2}} \quad (5.9)$$

Eliminating successively v and E between Eqns(5.7) and (5.8) gives the relations:

$$E^2 = m^2 V^4 + p^2 V^2 \quad (5.10)$$

$$v = \frac{pV^2}{(m^2 V^4 + p^2 V^2)^{\frac{1}{2}}} \quad (5.11)$$

⁵See Einstein’s discussion of the parallel velocity addition formula in Chapters VI and VII of Ref [1]

It follows from Eqn(5.11), that any massless object has velocity $v = V$ in any inertial frame. Identifying light with the massless photon then predicts that, in all inertial frames:

$$\text{speed of light} \equiv c = V = \text{constant} \quad (5.12)$$

so that also $\tilde{\gamma}_v = \gamma_v$ in all inertial frames. Eqn(5.12) is just Einstein's postulate E2 of special relativity. It follows directly from the identification of light with massless particles — for the discovery of which (not special relativity) Einstein was awarded the Nobel Prize.

The definition (5.2) and the identity (5.3) can be used to discuss the apparent velocities, in the embankment frame S, of the photons G_U and G_R that are detected by D' in the train T_U . Referring to Fig.5, the geometry of the configurations in Fig.5a and Fig.5c, Eqn(5.2) gives the relations:

$$v_{G_U D'} = \frac{c}{\gamma_v} \quad (5.13)$$

$$v_{R_U D'} = -\frac{c}{\gamma_v} \quad (5.14)$$

$$v_{G_U R_U} = \frac{2c}{\gamma_v} \quad (5.15)$$

Since $v_{D'D_U} = v$, Eqn(5.3) together with (5.13) and (5.14) can be used to derive the relations:

$$v_{G_U D_U} = c \left(\beta + \frac{1}{\gamma_v} \right) \geq c \quad (5.16)$$

$$v_{R_U D_U} = c \left(\beta - \frac{1}{\gamma_v} \right) \leq c \quad (5.17)$$

The maximum value of $v_{G_U D_U}$ or $\sqrt{2}c$ occurs when $\beta_v = 1/\gamma_v = 1/\sqrt{2}$. This is also the value of β_v for which $v_{R_U D_U}$ changes sign and the photon R_U apparently moves to the right, rather than the left in Fig.5. As $\beta \rightarrow 1$, $v_{G_U D_U} \simeq v_{R_U D_U} \rightarrow c$ so that the relative velocity of the photons G_U and R_U , as observed from S, vanishes. Comparison of the relative velocities (5.4) and (5.5) appropriate to Einstein's interpretation, uniquely in the frame S, of the TETE, with those (5.13) and (5.14) which describe instead the behaviour of the moving photon clock constituted by T_U , as observed from S, provides the correct understanding in SR of both the 'relativity of simultaneity' phenomenon and the meaning of 'velocity' in Eqn(5.1). The relativity of simultaneity effect describes different time ordering of the *same events* in different inertial frames, e.g. the events Y' and Y_U described in the previous section. In Einstein's interpretation of the TETE *different events*, $R'-D'$ (actually $R-D_R$), $G'-D'$ (actually $G-D_G$) and $G-R-D$ in the *same* inertial frame are compared. The xt -contiguity of $G-R-D$ in S (Fig.1c) and the apparent lack of it in S' (fig.3b and Fig.3d) are at variance with IC, which is a necessary consequence of the LT. In Einstein's interpretation of the TETE both the photons which are detected in D (at rest in the frame S) and apparently detected in D' (at rest in the frame S') have velocities, relative to S, of c . However D' in this case is not a detector *in* S' , it is instead identified with detectors in S, D_R and D_G , that happen to be at the same spatial positions in S as D' when photons are detected. Indeed, the photons detected in D' cannot be the the same ones as those detected in D_R or D_G . When the postulate E2

is correctly invoked, as in the analysis presented in Fig.5, it can be seen that the apparent velocities in S of the photons detected in D' in S', given by Eqns(5.13) and (5.14) are not, in general, equal to c , so that it is incorrect to calculate the times of coincidence of photons with D' by considering only photons with speed c in the frame S. This difference of apparent velocity is a consequence of the apparent slowing down of physical processes (in this case the speed of light propagation in an inertial frame) observed for all moving clocks as a consequence of the universal relativistic time dilatation effect⁶.

6 Comments on some previous interpretations of the Einstein TETE

A recent paper by Nelson [4] presented an extensive re-interpretation of Einstein's original TETE. The present author read this paper before writing the present one and found Nelson's analysis extremely useful in revealing the crucial questions which need to be addressed in the problem. Nelson finally concluded that Einstein's interpretation was qualitatively correct, in that the lightning strokes appear to be simultaneous to the observer on the embankment and non-simultaneous to an observer on the train, but that actually working out the problem quantitatively, in the embankment frame, according to Einstein's prescription, gave a result for the time difference of the non-simultaneous events different from that found in Nelson's 'relativistic' analysis, to be discussed below. This disagrees with the conclusion of the present paper that observers both on the embankment and in the train will judge that the strokes are simultaneous.

Although Nelson's paper contains many important and true statements concerning the problem, which will be discussed below, the reason for the difference, and why Nelson's (and Einstein's) conclusion is wrong becomes immediately obvious on performing a correct space-time analysis only of the lightning stroke events as observed on the embankment and in the train. Indeed, the light signals introduced by Einstein are quite irrelevant to the question of the simultaneity (or not) of the lightning stroke events themselves⁷.

According to Einstein's original formulation of the TETE: (i) the lightning strokes are simultaneous in the embankment frame. Einstein also states: (ii) 'But the events A and B also correspond to the points A and B on the train'. Denote the lightning stroke events in the embankment frame as E_A and E_B and in the train frame as $E_{A'}$ and $E_{B'}$. The simultaneity of $E_{A'}$ and $E_{B'}$ as a consequence of that of E_A and E_B follows directly from the relation (2.11) connecting corresponding time intervals in S and S'. If $\Delta\tau_A = \Delta\tau_B$, according to synchronised clocks in S, so that E_A and E_B are simultaneous in S

⁶A similar discussion of the apparent velocity of photons in the arms of a Michelson interferometer, considered as a 'photon clock', may be found in Ref. [8].

⁷In Chapter VIII of Ref [1] Einstein mixes up the physical definition of simultaneous events with the problem of how a single observer would decide, experimentally, that two events were simultaneous. In connection with the latter, light signals were introduced. This discussion leads directly to the introduction of the TETE in the following chapter. Simultaneity can be conveniently defined by conceptually simpler methods e.g. by establishing a network of pre-synchronised clocks and comparing event times recorded by local clocks. It was the misunderstanding of the physics of detection of the light signals, not of the lightning strokes, that lead Einstein to the erroneous conclusion of non-simultaneity of the train frame observations.

then also $\Delta t'_A = \Delta t'_B$ and $E_{A'}$ and $E_{B'}$ are simultaneous in S' . The lightning strokes events specified in the embankment frame are therefore simultaneous for observers both on the embankment and in the train as shown in Figs.1 and 3. This follows directly from Einstein's statements (i) and (ii) that define the initial conditions of the TETE, and the TD relation (2.11). This conclusion is, of course, independent of any subsequent analysis of light signals produced by the lightning strokes.

Because, unlike in the analysis presented above, using solely the coordinate-free TD relation (2.11), Nelson used the LT directly in his study of the TETE, the (coordinate-dependent) description of the experiment using the LT and synchronised clocks is now reviewed before coming to the critique of Nelson's arguments. The essential point here is that the LT is, fundamentally, not a description of the 'geometry of space-time' but of how similar clocks are observed to behave in different inertial frames⁸. The relation

$$\tau(E_A) = \tau(E_B) = t'(E_{A'}) = t'(E_{B'}) = 0, \quad (6.1)$$

as in the analogous TETE configuration shown in Fig.4, corresponds to a particular choice of the synchronisation convention of clocks in the frames S and S' . Choosing the origins of the coordinate systems in S and S' at M and M' , as done by Nelson, the synchronisation convention (6.1) corresponds to the following space-time transformations for clocks situated at A' , M' and B' respectively:

$$x'_{A'} + L/2 = \gamma_v[x_{A'} + L/2 - v\tau_A] = 0 \quad (6.2)$$

$$t'_{A'} = \gamma_v[\tau_A - \frac{v(x_A + L/2)}{c^2}] \quad (6.3)$$

$$x'_{M'} = \gamma_v[x_{M'} - v\tau_M] = 0 \quad (6.4)$$

$$t'_{M'} = \gamma_v[\tau_M - \frac{vx_{M'}}{c^2}] \quad (6.5)$$

$$x'_{B'} - L/2 = \gamma_v[x_{B'} - L/2 - v\tau_B] = 0 \quad (6.6)$$

$$t'_{B'} = \gamma_v[\tau_B - \frac{v(x_{B'} - L/2)}{c^2}] \quad (6.7)$$

Choosing $\tau_A = \tau_M = \tau_B \equiv \tau = 0$ when $x_{A'} = -L/2$, $x_{M'} = 0$ and $x_{B'} = L/2$, as in the configuration shown in Fig.4, gives $t'_{A'} = t'_{M'} = t'_{B'} \equiv t' = 0$, which is just the synchronisation convention (6.1) above. With this convention Eqns(6.2) to (6.7) simplify at S time⁹ τ to :

$$x'_{A'} = -L/2, \quad x_{A'} = -L/2 + v\tau, \quad t'_{A'} = \frac{\tau}{\gamma_v} \quad (6.8)$$

$$x'_{M'} = 0, \quad x_{M'} = v\tau, \quad t'_{M'} = \frac{\tau}{\gamma_v} \quad (6.9)$$

$$x'_{B'} = L/2, \quad x_{B'} = L/2 + v\tau, \quad t'_{B'} = \frac{\tau}{\gamma_v} \quad (6.10)$$

Thus

$$t'_{A'} = t'_{M'} = t'_{B'} = t' = \frac{\tau}{\gamma_v} \quad (6.11)$$

no 'relativity of simultaneity' and

$$x'_{B'} - x'_{A'} = x_{B'} - x_{A'} = L \quad (6.12)$$

⁸Just this remark was made by Einstein in Chapter X of Ref.[1].

⁹This is the time registered by all synchronised clocks in S .

no ‘length contraction’. The relations (6.11) and (6.12) were previously derived above (Eqn(2.11) and (4.2) respectively), without explicit use of the LT, by considering invariant space-time intervals in the frames S and S’.

After this preliminary discussion a survey of the arguments presented in Ref. [4] is made. The crucial statement in the abstract:

It is shown that, under the conditions of the experiment, a simultaneity in one inertial frame does indeed hold in another inertial frame, but it is perceived as a non-simultaneity by an observer outside the given frame.

is true and important for the understanding of the problem addressed, but its meaning depends crucially on that assigned to the word ‘perceived’. I return to this point below.

I agree with almost everything in Nelson’s Sections 2-4. Indeed, since Einstein’s analysis is carried out only in the embankment frame (there are no primed coordinates or times) nothing can be said concerning observations in the train frame. If the corresponding events are not even considered in the analysis it is evidently not possible to discuss simultaneity (or lack of it) in the train frame S’. To put it bluntly, Einstein’s analysis does not even address the question posed by the TETE. For this, coordinates and times in the train frame must be considered. This was clearly and correctly stated by Nelson in his Section 3. In fact the ‘non-simultaneity’ assigned by Einstein to the observations of the train observer has a trivial explanation —the different relative velocities of D’ relative to the photons G’ and R’ in the frame S (see Fig.1) ¹⁰, so, as explained above, what is shown in Fig.1 is the interaction of photons in the frame S (i.e. moving with the speed c in this frame, according to the postulate E2) with *detectors at rest in S at the same position as the moving detector D’ in this frame at the instant of photon detection*. It tells nothing of the response of D’ to photons that are moving, according to E2, with the speed c in the frame S’. This is what is shown in Fig.3. Here D is in motion in S’ with different velocities relative to the photons G and R and the ‘non-simultaneity’ of its apparent¹¹ encounters with them, is the same purely classical effect ¹², following from the definition of relative velocity in Eqn(5.2), as seen in Fig.1.

Nelson’s digression 4.1 is irrelevant to the discussion since, as shown above, the question of simultaneity or non-simultaneity of the lightning strokes in the frames S and S’ (before any consideration of light signals) has the same answer —yes they are simultaneous in both frames— whether a Galilean or Lorentz transformation is used. The simple reason is that the effect of the LT must be the same at the points A and B, — this is just translational invariance [8]. So if $\tau_A = \tau_B$ then necessarily $t'_A = t'_B$. If the same synchronisation convention ¹³ is used at A and B this will always be the case. Thus the statement in Section 4 of Ref. [4]:

¹⁰In discussing points raised in Ref. [4] the notation of the present paper is used

¹¹As in S, the corresponding S’ frame events are actually specified, as discussed Section 3 above, by the coincidences of photons with detectors at rest in S’ at the same positions as the moving detector D at the S’ times of the coincidences.

¹²It is classical because precisely the same result is obtained in Galilean relativity, where photons are replaced by massive objects of constant and equal velocities. The same pattern of events is seen in Fig.2 where light signals are replaced by sound signals. The underlying physical description of sound propagation is also purely classical.

¹³That is, the initial settings τ_0 and t'_0 , as viewed from S, of clocks at rest in S and S’ respectively.

Put another way $A(\text{embankment})$ and $A(\text{train})$ are space-time coincident ---i.e. they are one and the same point--- when the strokes occur, so that a stroke that hits $A(\text{embankment})$ also hits $A(\text{train})$ at the same instant. Likewise the stroke hitting $B(\text{embankment})$ also hits $B(\text{train})$.

is true in both Galilean and special relativity, on twice replacing ‘also hits’ by ‘is also observed from’. It is essential only that the strokes ‘hit’ the embankment, not also the train. This was the way Einstein’s TETE was defined in Ref[1]. Indeed (see the Appendix) depending of how precisely the ‘strokes’ are defined, they may not even hit the ends of the train at any time. Also, as shown above, ‘the train spans the same length in each frame’ — there is no length contraction. Thus Nelson’s remarks in Section 4.1 on the ‘pre-relativistic’ nature of the analysis presented later in Section 7 of Ref. [4] are ill-founded. Nelson’s Figs.5 and 6 describe correctly the problem in both Galilean and special relativity. Only the physical meaning of the photon-detector coincidence events (in the language of the present paper) is misinterpreted by Nelson.

I find the ‘Traditional’ analysis of Section 6 of Ref. [4] to be quite spurious. Non-simultaneous events observed in the frame S due to different photon-detector relative velocities are transformed into the frame S' and interpreted in terms of photons observed in this frame (i.e. photons with speed c in S'). I can see no physical rationale for such a procedure.

The analysis presented in Sections 7 and 8 of Ref. [4] is very similar to that in Section 3 of the present paper. Figures 5 and 6 of Ref. [4] are essentially the same as Figs.1 and 3 of the present paper, the same photon-detector coincidence times being found. In Section 7 is found the statement: ‘This would seem to show that simultaneity in one frame does indeed hold in another frame, but does not appear this way to an observer outside the frame’. This point is the same as that quoted above from the abstract of Ref. [4]. The photons G and R are detected simultaneously in S by D , whereas the other photons G' and R' apparently encounter the detector D' at different times in S (Fig.1) On the other hand the photons G' and R' are detected simultaneously by D' in S' whereas the photons G and R apparently encounter the detector D at different times in S' (Fig.3). The phrase ‘does not appear this way to an observer outside the frame’ refers, for example, to the S' frame observation of temporally distinct G - D and R - D coincidences instead of the triple G - R - D coincidence observed in S . But of course, as explained above *these are not the same events*. The photons G and R detected in S are not the photons G and R detected in S' . Also the moving detector D in S' is actually strictly equivalent to detectors *at rest in this frame* which happen to be at the same spatial position as the detector D at the instant at which the photons are detected. Only in this case can a meaningful time (that in S) be assigned to the corresponding events.

The pattern of events shown in Figs.1 and 3 is, as previously stated, consistent with the Measurement Reciprocity Postulate. Contrary to what is stated in Section 7 of Ref. [4] there is no paradox to be resolved here. What are shown in Table 1 of Ref. [4] are not inconsistent observations of the lightning strokes but the predicted and compatible time differences of *different* photon-detector coincidence events

In Section 9.1 of Ref. [4] the important remark is made that space-time coincidences of different events are invariant and so their existence must be agreed upon by observers

Event	R-D _R	G-R-D	G-D _G	G'-D _G '	G'-R'-D'	R'-D _R '
Time in S, τ	$\frac{L}{2(c+v)}$	$\frac{L}{2c}$	$\frac{L}{2(c-v)}$	$\frac{\gamma_v L}{2(c+v)}$	$\frac{\gamma_v L}{2c}$	$\frac{\gamma_v L}{2(c-v)}$
Time in S', τ'	$\frac{\gamma_v L}{2(c+v)}$	$\frac{\gamma_v L}{2c}$	$\frac{\gamma_v L}{2(c-v)}$	$\frac{L}{2(c+v)}$	$\frac{L}{2c}$	$\frac{L}{2(c-v)}$

Table 1: *Observation times on the embankment (frame S) and in the train (frame S') of the six distinct photon-detector coincidence events.*

in any frame. — this is just the ‘Invariance of Contiguity’ proved in Section 2 above. It is suggested (as is done in the refined TETE of Section 4 of the present paper) to generate a third light signal if a double light signal-detector coincidence is recorded (for example the G’-R’-D’ coincidence of Fig.3 of the present paper). Of course, as is shown in Fig.5 (which is analogous to Fig.1 for the original TETE) the embankment observer *does* see the signal Y’ that shows that the analogous G_U -R_U -D’ coincidence does occur in the train. Similarly, as shown in Fig.6, the train frame observer sees the Y_U signal indicating the G’-R’-D_U coincidence on the embankment which is analogous to the G-R-D one of Fig.1. Contrary to what is said in Section 9.1 of Ref. [4] there is no ‘repudiation’ of invariants. The mistake here is the identification of the separate R’-D’ and G’-D’ coincidence events of Fig.1b and 1d respectively in the frame S with the triple coincidence event G’-R’-D’ of Fig.3c in the frame S’. The photons R’ and G’ of Fig.1, that are detected in S are not the same as R’ and G’ in Fig.3, observed in coincidence in the frame S’, The three coincidence events R’-D’-not G’ (Actually R-D_R), G’-D’-not R’ (Actually G-D_G) in S and G’-R’-D’ in S’ all occur, *but they are all distinct events*, not the same events as viewed in different frames, as assumed by Nelson, so that Invariance of Contiguity is respected..

Contrary then to what is stated in Section 10 of Ref [4] the patterns of events shown in Figs.5 and 6 of Ref [4] or in Figs.1 and 3 of the present paper are perfectly consistent. There are three distinct events for observers in either the embankment or the train frames: two single photon-detector coincidences and one double photon-detector coincidence, giving, in total, *six different events* seen directly in the embankment and train frames. The fundamental error of both Einstein and Nelson is the attempt to analyse the problem in terms of only three events specified only in the embankment frame. The mistake made here is similar to that, mentioned in Section 2 above, where it is attempted to analyse the TD effects for observers in the frames S and S’ uniquely in terms of $\Delta\tau$ and $\Delta\tau'$ instead of the four physically distinct time intervals $\Delta\tau$, Δt , $\Delta\tau'$ and $\Delta t'$.

The three coincidence events observed in each of the embankment and train frames, may, if light signals are also generated by the single photon-detector coincidences (as described in Section 5 above for the double coincidence events) be observed from the other frames. This gives, in each frame, six events: the three coincidences recorded by the detectors at rest in the frame, and three signals produced by the similar coincidence events in the other frame —in all twelve distinct events instead of the three assumed by Einstein and Nelson. It should be obvious that all these events are distinct. Six correspond to ‘baking a cake’, three on the embankment and three on the train while six correspond to ‘watching the cake bake’, again three on the embankment and three on the train. Of course the cakes being baked on the embankment and in the train are quite distinct. The times of all twelve events are presented in Table 1.

In Nelson’s ‘relativistic analysis’ presented in Section 11 and the appendix of Ref [4],

as in Einstein's analysis, only events defined in the embankment frame are considered, but unlike Einstein, the LT is used to transform these events into the train frame. In the notation of the present paper, the events $R-D_R$ and $G-D_G$ are identified with events $R'-D'$ and $G'-R'$, respectively in the train frame and it is concluded, by Lorentz transforming these events into the train frame, that there is no $R'-G'-D'$ coincidence event in the latter frame. A complicated calculation is then performed transforming the lightning stroke events from S into S' , as well as finding the propagation delays between light signals in S' produced by the transformed stroke events and the moving detector D in this frame, to show that the double coincidence event $R-G-D$ in S is also observed as such by the train observer. Actually, this follows directly from the application of IC to this event without any consideration of the prior lightning strokes and photon propagation. The spurious 'relativity of simultaneity' effect given by incorrect use of the LT (6.4) and (6.5) appropriate to M and M' to transform the lightning stroke events, (instead of (6.2) and (6.3) for the stroke at A and (6.6) and (6.7) for the stroke at B), predicts that the lightning strokes will be seen with a time difference $\Delta\tau'_{RS} = \gamma_v v L/c^2$ by the train observer. Nelson claims that this time difference is equal to that between the $R-D_R$ and $G-D_G$ coincidences (wrongly identified with $R'-D'$ and $G'-D'$ coincidences observed by the train observer). Taking into account the TD effect of Eqn(2.12) the train observer will see, from the entries of Table 1, a time difference between these events of:

$$\Delta\tau'_{TD} = \frac{\gamma_v L}{2} \left[\frac{1}{c-v} - \frac{1}{c+v} \right] = \frac{\gamma_v^3 L v}{c^2} \quad (6.13)$$

In order to get agreement between $\Delta\tau'_{RS}$ and $\Delta\tau'_{TD}$ Nelson applied the TD effect in the opposite direction, i.e. the formula $\Delta t = \gamma_v \Delta\tau'$ was used instead of the correct formula (2.12). Thus, by dint of not considering events in the train frame, invoking the spurious 'relativity of simultaneity' effect and an incorrect calculation of TD, Nelson arrived at a result in qualitative agreement with Einstein's one.

To summarise Nelson's paper: It was correctly pointed out that Einstein's interpretation, considering events only in the embankment frame says nothing about what an observer on the train will see, as events in the train frame are not even considered. Assuming simultaneous lightning strokes on the embankment and in the train, and no length contraction (stated by Nelson to correspond to a 'pre-relativistic' analysis, but actually the correct prediction of both Galilean and special relativity) the similar sequences of events on the embankment and in the train are then considered (Section 7 and Figs. 5 and 6 of Ref. [4]). From these it is correctly concluded that both embankment and train observers see simultaneous events. It is then assumed (in the notation of the present paper) that the photon-detector coincidences $R-D_R$, (called $R'-D'$) and $G-D_G$ (called $G'-D'$) and $R'-G'-D'$ are different observations of the same event: $R'-G'-D'$ in S' and $R'-D'$ and $G'-D'$ in S . This is in clearly in contradiction with Invariance of Contiguity, and so Nelson concludes that the $R'-G'-D'$ coincidence in S' does not exist. In view of the reciprocal nature of the events shown in Figs.5 and 6 of Ref. [4], it could have been argued, by the same logic, that the $R-G-D$ coincidence in S does not exist. The fallacy of such arguments is, for example, the incorrect identification of $G-D_G$ with $G'-D'$ and $R-D_R$ with $R'-D'$. Actually $G-D_G$ and $R-D_R$ in S and $R'-G'-D'$ in S' are distinct and uncorrelated events (different photons are detected in each case). The 'relativistic' solution suggested has been discussed above. Only events in the embankment frame are considered, the spurious 'relativity of simultaneity' effect, resulting from misuse of the LT, is invoked, and the TD

effect is incorrectly calculated, to yield an interpretation in qualitative agreement with Einstein's one.

Nelson's correct insistence on the necessity of analysing the problem in both the train and embankment frames was contested in Comments by Rowland [19] and Mallinckrodt [20]. Unlike Nelson, no attempt at a quantitative analysis of the problem was proposed, so that only verbal arguments were presented. Rowland correctly stresses the importance of Invariance of Contiguity for the problem but wrongly concludes from this that embankment frame analysis only, as in Einstein's interpretation, is sufficient. Only embankment frame events are discussed, so the essential problem is not addressed. Similarly, Mallinckrodt considers only events in the embankment frame but makes the additional and incorrect statement (rebutted in Nelson's reply [21] to the Comments) that the postulate E2 is essential for Einstein's interpretation. None of Nelson, Rowland and Mallinckrodt understand the essential point that *different events* (i.e. events initiated by different photons) are observed in the embankment and train frames, so that it is fallacious to assume that they are the same events viewed in different frames. This fact is a consequence of relativistic kinematics, one important aspect of which is encapsulated in Einstein's postulate E2. If the photons in the TETE were replaced by massive particles obeying Galilean kinematics Einstein's interpretation would be correct and the three events $R-D_R$, $G-D_G$ and $G-R-D$ would be observed at the same time in both the train and embankment frames and would constitute a complete and correct description of the problem. But one would not conclude from this, any more than for the sound signals in Fig.2, that 'relativity of simultaneity' an essential feature of Galilean Relativity or of acoustics.

The essential point, that it is not possible, in special relativity, to associate the times of coincidence events in detectors at rest in a reference frame with the proper time of an event in a spatially contiguous but moving detector is made particularly clear in the example discussed below in the Appendix in which the 'lightning strokes' of Einstein's TETE are replaced by two pulsed laser beams in the embankment frame.

7 Summary and conclusions

In Einstein's TETE the lightning strokes at A and B (see Fig.1a) are, by definition, simultaneous in the embankment frame. It then follows directly from the TD formula (2.11) that these events are simultaneous in the train frame, i.e. if $\Delta\tau_A = \Delta\tau_B$ in (2.11) then necessarily $\Delta t'_A = \Delta t'_B$. This is exemplified by comparison of Fig.8b and Fig.12 in the Appendix —the events corresponding to the photon bunches striking the points A and B are simultaneous in both the embankment and the train frames. However, in order to analyse the problem, Einstein did not consider directly the lightning stroke events but rather light signals produced by the strokes and later seen by observers at the middle of the train or at point on the embankment midway between A and B. Considering only events in the embankment frame, as shown in Fig.1, it was concluded that the embankment observer would see the signals from the lightning strokes simultaneously (Fig.1c) but not the train observer (Fig.1b and Fig.1d). The latter then infers that the lightning strokes must occur at different times in the train frame. Einstein took this inference of the

supposed observations of the train observer as a statement of fact about any two events in an inertial frame [1]:

Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train and vice versa (relativity of simultaneity).

This interpretation is based on the assumption that the times of events in the train frame may be identified with the times of events observed in the embankment frame at the same apparent spatial positions. For example, in Fig.1b, the time of the train frame coincidence event $R'-D'$ is assumed to be the same as the embankment frame event $R-D_R$. As shown by the calculation presented in the Appendix, this would be true if the photons in Fig.1 would be replaced by massive particles obeying Newtonian kinematics. However this assumption is false (and therefore invalidates Einstein's argument) for massive or massless particles obeying relativistic kinematics. Fundamentally, it is this neglect of relativistic kinematics, embodied, for photons, by the postulate E2, that leads Einstein to a conclusion that is the opposite of the correct one. Using E2, and performing the analysis in the train frame, as shown in Fig.3, leads, in combination with the embankment frame analysis of Fig.1, to the correct conclusion that the lightning strokes are observed to be simultaneous by both the embankment and the train observers.

As shown by the analysis of the TETE with two trains presented in Section 4, a genuine 'relativity of simultaneity' effect does exist, in special relativity, for certain events, in *different* inertial frames. However, as shown above, it follows directly from the TD effect formula that two events which are simultaneous in the *same* inertial frame are so in all such frames —contrary to Einstein's statement quoted above.

In Section 5, the precise operational meaning of the postulate E2 is discussed and the important distinction between 'speed' and 'relative velocity' is pointed out. It is also shown that E2 does not need to be introduced as a separate (and anti-intuitive) hypothesis, in special relativity but is direct consequence of the relativistic kinematics of massless photons [7]. Derivations of the LT not using the postulate E2 are briefly mentioned in Section 2 where the TD effect and Invariance of Contiguity are introduced. Due to the TD effect, the apparent speed of light signals between events, observed in a moving inertial frame from another one, may be greater than, or less than, the light speed, c , as measured in an inertial frame (Eqns(5.16) and (5.17)).

Section 6 contains a detailed discussion of Nelson's [4] re-interpretation of Einstein's TETE. Nelson correctly pointed out the impossibility to analyse the problem in a meaningful manner by considering only events in the embankment frame. The correct solution of the problem was presented in Nelson's Section 7 and shown in his Figs.5 and 6 (the latter being equivalent to Figs.1 and 3 of the present paper). However, this solution was rejected by the failure to notice the distinct nature of (in the language of the present paper) the photon-detector coincidence events in the different frames. See Table 1 and the accompanying discussion in the text. These were misinterpreted by Nelson as observations, in different frames, of the same events, with a resulting apparent 'repudiation of invariants' or violation of Invariance of Contiguity. This lead to the rejection of the correct interpretation shown in Figs.5 and 6 of Ref. [4] and the introduction of the 'relativistic analysis' of Nelson's Section 11 and Appendix. This, like Einstein's interpretation, consid-

ered only coincidence events in the embankment frame, which were, however, subsequently Lorentz-transformed into the train frame. In the calculation the spurious ‘relativity of simultaneity’ effect (correlated with the equally spurious ‘relativistic length contraction’ effect) resulting from misuse of the space-time LT [8, 9, 10, 11] was invoked, and the TD effect of Eqn(2.12) between the train and embankment frames was inverted. This incorrect calculation gave a result in qualitative agreement with Einstein’s interpretation.

In the Appendix of the present paper, instead of considering observations of light signals produced by the ‘lightning strokes’ of Einstein’s TETE, the latter are replaced by temporally localised bunches of laser photons generated in the embankment frame (see Fig.8a). Applying Einstein’s interpretation to the configuration of Fig.8b, where the bunches of photons hit the points A and B on the embankment, it would be concluded that the former hit simultaneously the xt -contiguous points A’ and B’ on the train. The detailed space-time analysis of the problem presented shows that this would indeed be the case if the photons were replaced by massive particles moving at a constant velocity u , $\ll c$, but this is no longer the case when u is of the order of c . Comparison of Figs.10, 11 and 12 shows the profoundly non-intuitive nature of the spatial configurations of moving objects at corresponding times (i.e. times linked by the TD formulae (2.11) or (2.12)), in different frames, as predicted by special relativity theory.

Added Note

The conclusion of this paper, contrary to Einstein’s assertion, that observers both on the embankment and in the train will find that the lightning strikes on the embankment occur simultaneously is a valid one, but the arguments given to reach this conclusion are flawed by a fundamental conceptual error. Indeed, all the necessary information to obtain the correct solution is presented in Sections 2 and 5 together with the relation $L = L'$ that is, however, (see below) incorrectly derived in (4.2).

The error which is made (in common with all previous analyses of the problem, including that of Ref. [4]) is the conflation of kinematical configurations in a primary experiment and its reciprocal, related by the MRP as discussed in Section 2, with observations in the frames S or S’ of either one of these experiments which, as correctly stated in Section 2, are *physically independent*. In the primary experiment, clocks at rest in S’ are seen to run slower than clocks in S (Eqn(2.11)) whereas in the reciprocal experiment, clocks at rest in S are seen to run slower than clocks in S’ (Eqn(2.12)). As pointed out in connection with Eqns(2.13) and (2.14) it is impossible that clocks can run both fast and slow in the same experiment.

Since Einstein described the observations of coincidence events (between the world lines of observers and light signals) only in the embankment frame, these are the events that must be transformed into the train frame in order to analyse the experiment. Einstein did not do this and, as inspection of Figs. 5, 6 and 7 shows, neither did the present author! What are correctly shown in these figures are instead configurations of physically-independent primary and reciprocal longitudinal photon-clock experiments as previously discussed in connection with the Michelson Interferometer in Ref. [8]. The same mistake, arising from misinterpretation of the postulate E2, occurs in Fig. 4 of Ref. [4]. If the observed speed of light actually were c , independently of the speed of its source, in all inertial frames, as naively follows from the postulate E2, then the description of the

signals G' and R' shown in Fig. 5 is correct. However the signals G_U and R_U which, by definition, have speed c in the frame S' have, as discussed in Section 5, the observed speeds in S : $c_{app}^{\pm} = c(\beta \pm 1/\gamma_v)$ (Eqns(5.16) and (5.17)). As the Relativistic Relative Velocity Transformation Relation (RRVTR) of which these formulae are examples, is a necessary consequence of time dilatation and the invariance of length intervals, it generalises to $u = v + u'/\gamma_v$ where u and u' are the observed velocities of an object in S and S' respectively, *in the same space-time experiment*. Transposing this equation gives $u' = \gamma_v(u - v)$, which shows that the observed velocity of an object in S' is γ_v times the relative velocity of the object and the frame S' in the frame S . Since the detector D_U in Figs. 5 and has $u = 0$ it follows that its speed in the frame S' is not v , as shown in Fig. 6, but instead $\gamma_v v$. What are actually shown in Fig. 6 are, not, as claimed in the paper, the same events shown in Fig. 5, but viewed in S' , but rather the kinematical configurations of the *physically-independent reciprocal experiment*. Indeed the distinct and different TD effects according to Eqn(2.11) and (2.13) are evident in Fig. 5c and 6c respectively.

The ‘relativity of simultaneity’ effects discussed in connection with Figs. 5, 6 and 7 are therefore spurious, since the times which are compared are not observations of the same events in different frames in the same space-time experiment, but rather observations of events in the three *physically-independent* experiments that are shown in the figures. In the primary photon-clock experiment shown in Fig. 5, clocks at rest in S' run slower than clocks at rest in S , whereas in the reciprocal experiment shown in Fig. 6 clocks at rest in S run slower than clocks at rest in S' . Because of the symmetry of the configurations in Fig. 7, clocks at rest in S and S' run slower, by the same amount, as clocks at the rest in S'' .

The correct analysis of Einstein’s TETE on the basis of the non-simultaneous events in S and S' (which appear nowhere in Figs. 5, 6 and 7) according to the RRVTR is presented in Ref. [22]. These events transform into non-simultaneous events in the frame S' , but with prior knowledge of the parameters c and v and of the RRVTR it may be deduced from the observed times of the non-simultaneous light signal events that the lightning-stroke events, that are the sources of the light signals, are simultaneous in the train frame S' contrary to Einstein’s conclusion.

With the exception of the assertion of the correctness of Nelson’s Fig. 4 as corresponding configurations in the frames S and S' in the experiment (v in (b) should be replaced by $\gamma_v v$) the criticisms of Ref. [4] remain valid —the assertion, in the latter, in agreement with Einstein, of non-simultaneity of the lightning-stroke events in the train frame is indeed erroneous.

The same mistake concerning the physical interpretation of the postulate E2 as in Figs. 5-7 occurs in the analysis of the experiment presented in the Appendix. In Fig. 8b the world lines of A , A' and a light signal intersect in the same point as, simultaneously, do the world lines of B , B' and another light signal. As correctly stated in Section 2, the same coincidence events must be seen to occur, and to occur simultaneously, in all other inertial frames, in particular in that, S' , of the train. Inspection of Figs. 10 and 11 shows that this is not the case for the calculation presented in the Appendix where it is assumed, according to the naive interpretation of the postulate E2, that the light signals have the same speed c in both S and S' in the same space-time experiment. Use of the

RRVTR to correctly transform the photon velocities into the frame S' gives, correctly, the two simultaneous triple world intersections, but implies that the observed angles of the light signals in S' are not correctly given by the kinematical transformation formula (A.3) used in the Appendix to calculate the directions of the light signals. This important point will be further discussed in a forthcoming article.

The derivation of the relation $L = L'$ of Eqn(4.2) is wrong because it assumes that events on the world lines of the spatially separated points A' and B' both transform according to the ‘generic’ LT (2.1)-(2.4). Using the correct transformation equations, (6.2) and (6.3) for a synchronised clock at A' as well as (6.6) and (6.7) for one at B' , and setting L to L' on the left sides of (6.2) and (6.6), gives, instead of (4.1),

$$\begin{aligned} (\Delta x')^2 - c^2(\Delta t')^2 &= (\Delta x)^2 - c^2(\Delta \tau)^2 \\ &\quad + 2\gamma_v L'(\Delta x - L) - 2\Delta x L - 2\gamma_v v \Delta \tau + L^2 + (L')^2 \end{aligned} \quad (7.1)$$

Setting $\Delta t' = \Delta \tau = 0$ and $\Delta x = L$ in this equation leads not to the relation $L = L'$ but to the identity

$$\Delta x' \equiv L' = L' \quad (7.2)$$

In fact the equality of L and L' is established in a straightforward manner from (6.2) on replacing L by L' on the left side:

$$x'_{A'} + \frac{L'}{2} = \gamma[x_{A'} + \frac{L}{2} - v\tau_A] = 0 \quad (7.3)$$

Since L and L' in (20) are independent of v , this equation is valid for all values of v , in particular for $v = 0$, $\gamma = 1$ and $x \rightarrow x'$:

$$x'_{A'} + \frac{L'}{2} = x'_{A'} + \frac{L}{2} \quad (7.4)$$

or

$$L' = L \quad (7.5)$$

Appendix

In the analysis above of Einstein’s TETE, light signals produced by lightning strokes hitting the embankment were considered, as viewed by observers on the embankment or in the moving train. In this appendix the ‘lightning strokes’ are replaced light pulses, generated by a source in the embankment frame, that strike the embankment at points with the same x -coordinates as the ends of the moving train at this instant. The events corresponding to the detection of these light pulses, —analogous to the lightning stroke events in the Einstein TETE— are analysed directly rather than the light signals produced by, say, reflection of the pulses at the embankment, as in the Einstein TETE.

A pulsed laser source (PLS) produces two bunches of photons directed towards the points A and B on the embankment (Fig.8a). The timing of the laser pulses is such that they arrive at the points A and B at the instant that the ends A' and B' of the train are aligned with A and B (Fig.8b). With parameters θ and L as defined in Fig.8, this implies that the photon pulses arrive simultaneously at A and B at the time $\tau =$

$L/(2c \cos \theta)$, if they are emitted from the PLS at $\tau = 0$. The configurations shown in Fig.8 correspond $\theta = 60^\circ$ and $\beta_v = 0.5$. Applying Einstein's interpretation of his TETE to this experiment it would be concluded that the photon pulses are observed simultaneously in both the embankment or the train frames. It will now be demonstrated that this conclusion, which is correct if the photon pulses are replaced by massive objects obeying Newtonian kinematics, is no longer true in special relativity, whether or not massive objects or massless photons are considered. To demonstrate this, the following relativistic calculations will be performed assuming massive particles rather than photons. Newtonian kinematics and the case of photons are given by taking the $c \rightarrow \infty$ and $m \rightarrow 0$ limits, respectively, of the relativistic equations for particles with an arbitrary Newtonian mass m .

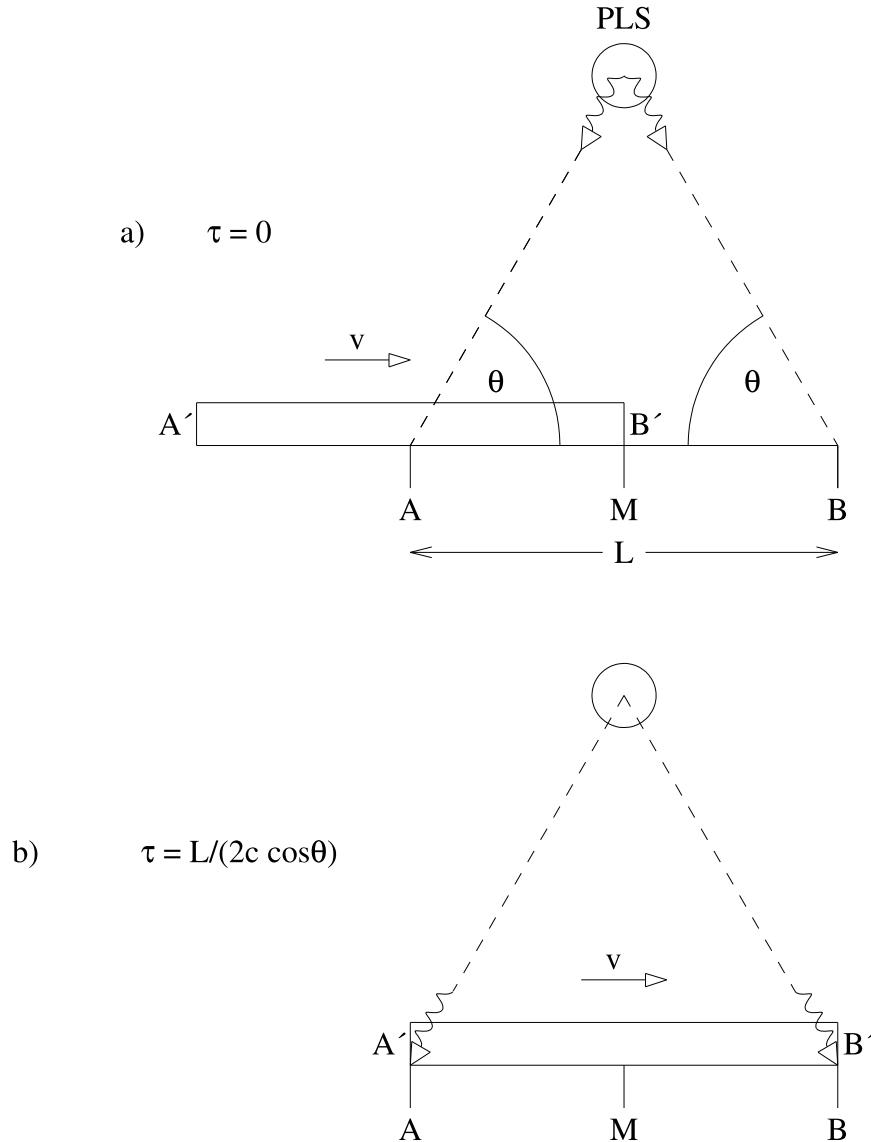


Figure 8: a) The pulsed laser source, PLS, produces, simultaneously, two bunches of photons directed towards the points A and B on the embankment at $\tau = 0$. b) at time $\tau = L/(2c \cos \theta)$ the photon bunches arrive at A and B at the instant that the points A' and B' on the train are xt-contiguous with A and B respectively. In this figure $\beta_v = 0.5$.

Transforming the photon momenta into the rest frame of the train at $\tau = \tau' = 0$

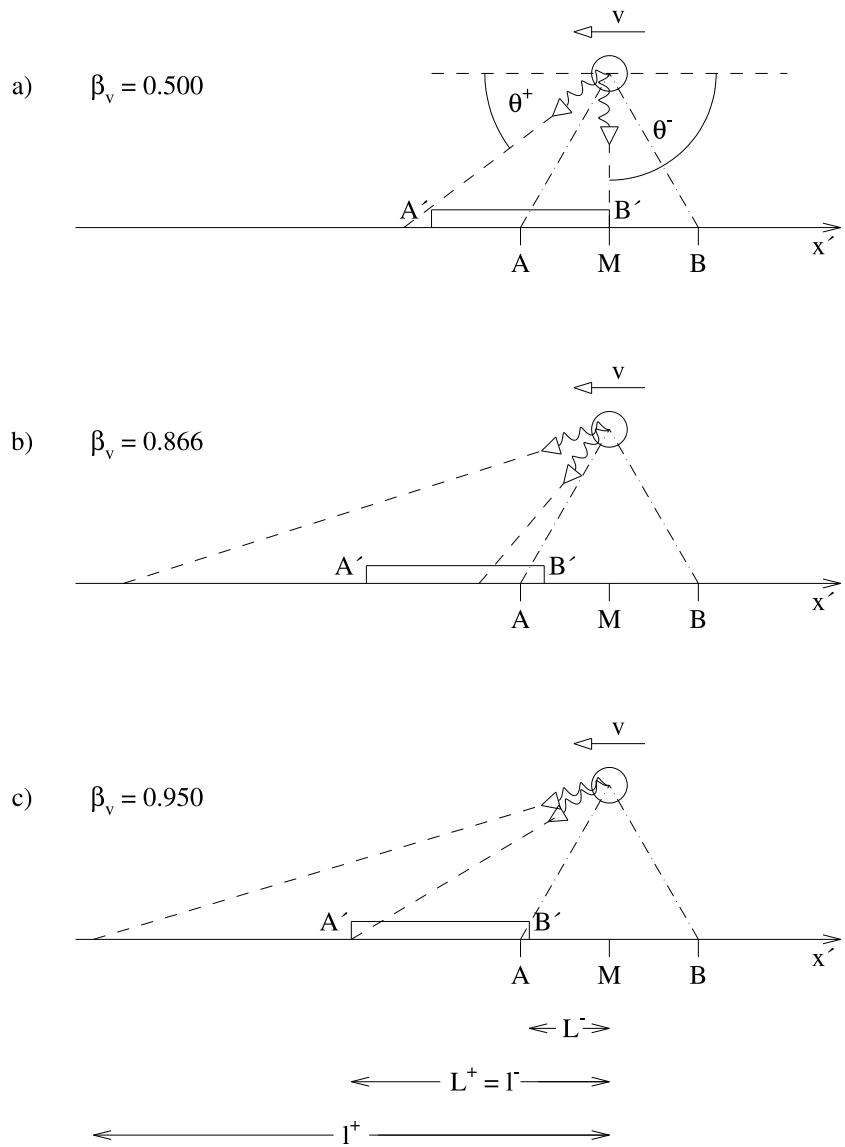


Figure 9: Directions and positions of the photon bunches from the PLS in the train frame at $\tau = \tau' = 0$ for various values of β_v .

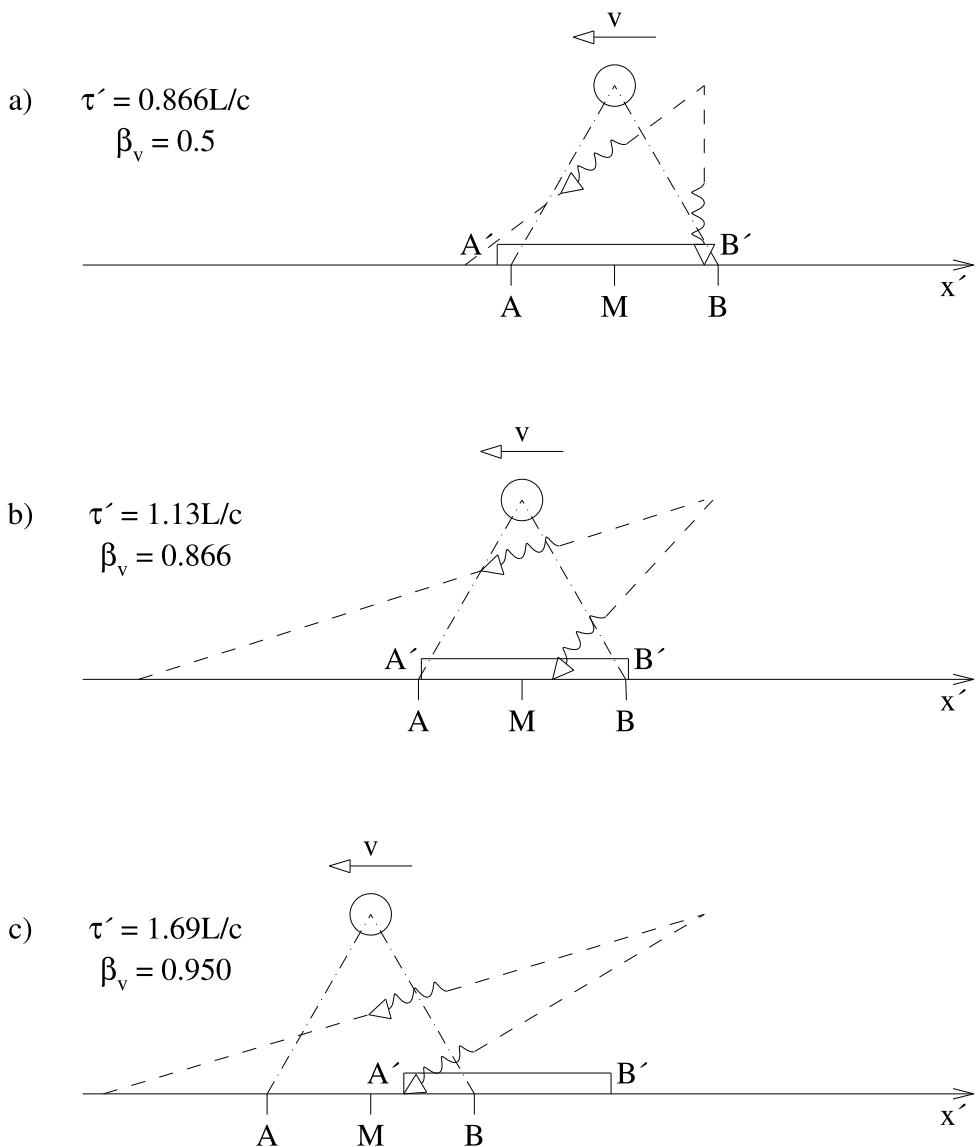


Figure 10: Positions of the photon bunches in the train frame at later times. a) $\beta_v = 0.5$, photon bunch xt -contiguous with B' ; b) $\beta_v = 0.866$, photon bunch crosses x' -axis; c) $\beta_v = 0.950$, photon bunch xt -contiguous with A' .

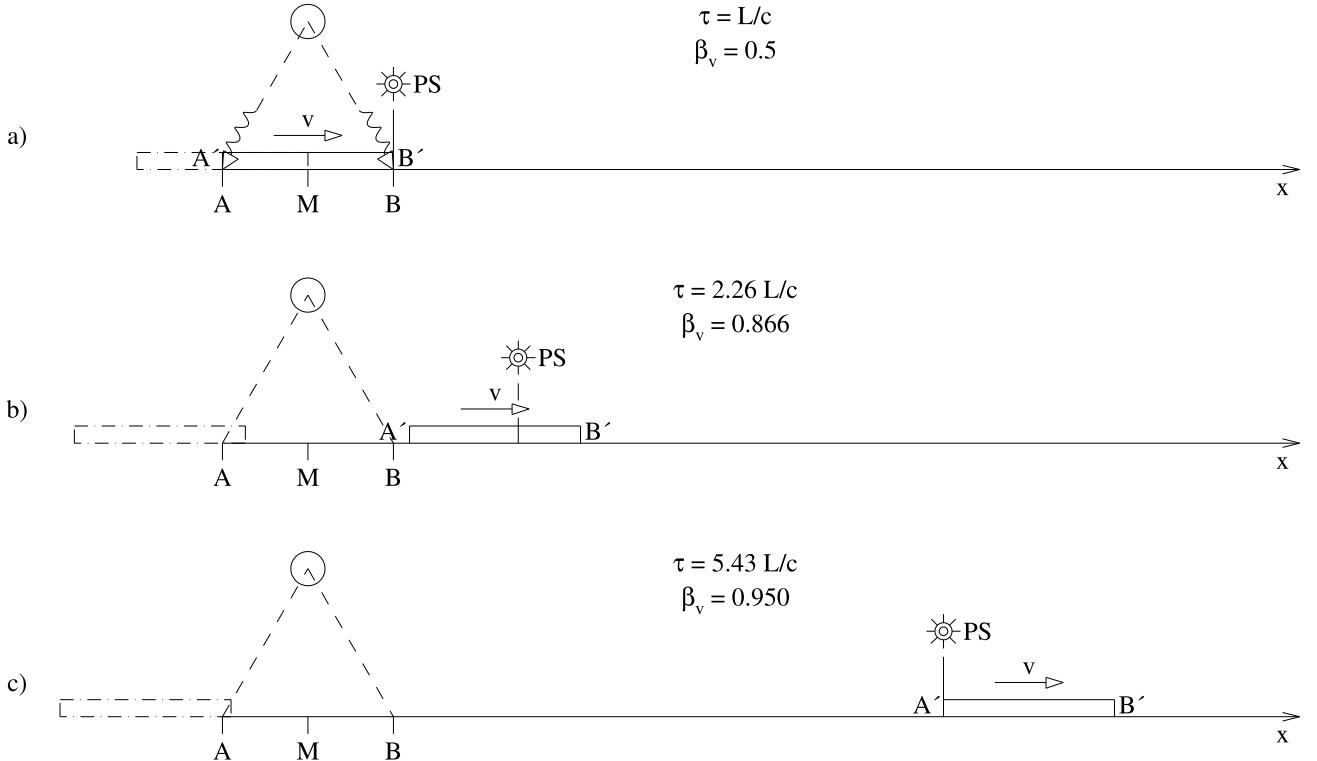


Figure 11: The train-frame xt -contiguous events shown in Fig.10 as observed from the embankment frame. Coincidences registered by detectors in the train produce the prompt signals, PS, shown. The positions of the train at $\tau = 0$ are shown dot-dashed.

gives, for the photon case, the kinematical and spatial configurations shown in Fig.9 for $v = c/2$, $\sqrt{3}c/2$ and $0.950c$. The rationale for the choice of the last value of v is explained below. Shown in Fig.9, in each case, are the positions of the train and of the points A, M and B on the embankment defined in the same way as in Einstein's TETE, as well as the positions and directions of flight of the photon bunches ¹⁴.

The latter are derived from the LT of relativistic momentum according to the equations:

$$p^\pm \cos \theta^\pm = \gamma_v (p \cos \theta \pm \frac{\beta_v E}{c}) \quad (\text{A.1})$$

$$p^\pm \sin \theta^\pm = p \sin \theta \quad (\text{A.2})$$

Eqn(A.1) is the transformation equation for the longitudinal component of momentum, while Eqn(A.2) expresses conservation of transverse momentum. Eqns(A.1) and (A.2), together with the relations $E = \gamma_u mc^2$ and $p = \gamma_u \beta_u mc$, where the particle of mass m is assumed to move with speed u , then give:

$$\tan \theta^\pm = \frac{u \sin \theta}{\gamma_v (u \cos \theta \pm v)} \quad (\text{A.3})$$

The angles θ and θ^\pm are defined on Fig.8 and Fig.9a respectively. If L^+ and L^- are the (positive) distances of A' and B' , respectively, from M and l^\pm are the (positive) distances

¹⁴In Figs. 8a and 9, the the photon bunches are located at the tails of the wavy arrows, in Figs.8b and 10-12 at the tips of the arrows.

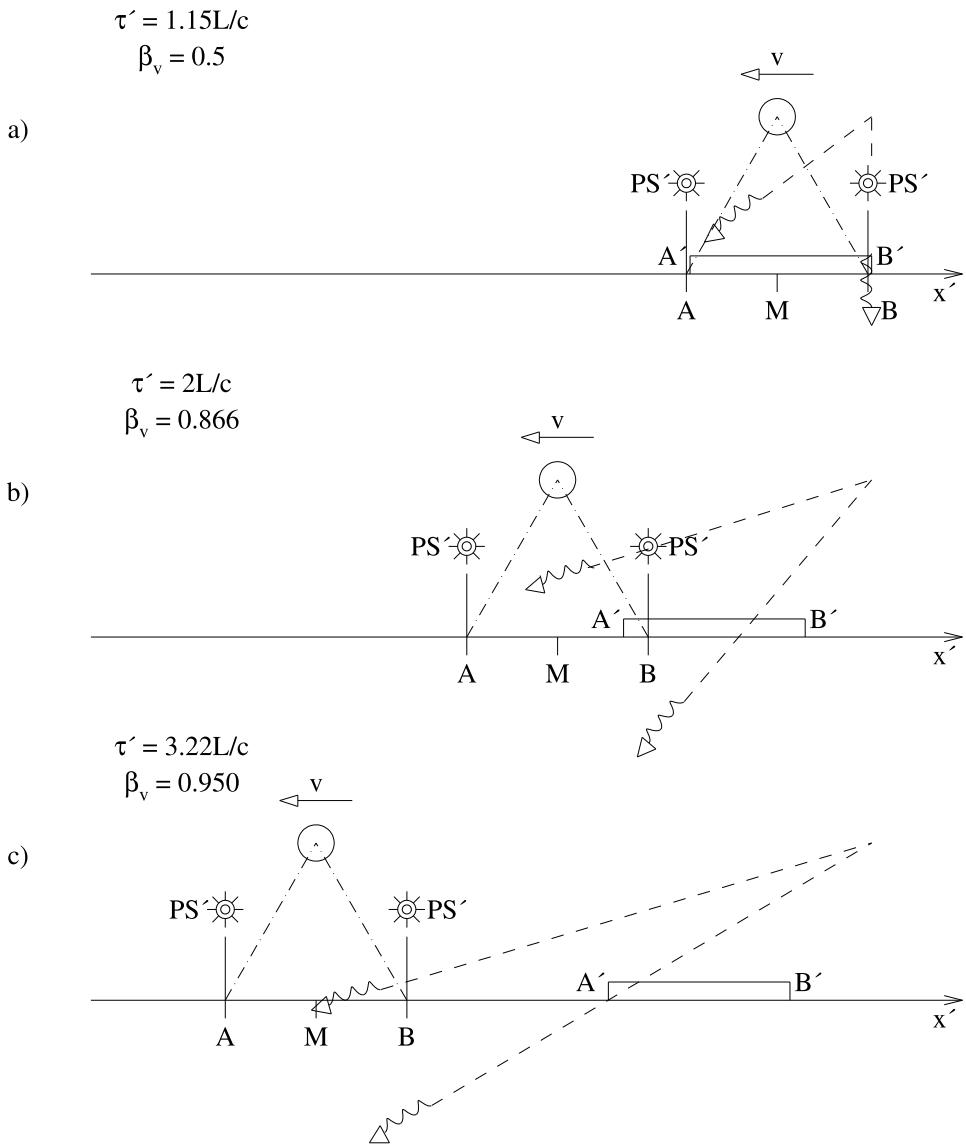


Figure 12: The simultaneous embankment frame photon-A and photon-B xt -contiguous events of Fig.8b, as viewed from the train frame, showing also the corresponding positions of the photon bunches in this frame. The photon-A and photon-B coincidence events in the embankment frame produce prompt signals, PS' , shown. These events are also simultaneous in the train frame. In this case there is no ‘relativity of simultaneity’.

from M of the intersections of the particle trajectories with the x' -axis (see Fig.9c), then the geometry of Fig.9 gives:

$$L^\pm = \frac{L}{2} \left(\frac{v}{u \cos \theta} \pm 1 \right) \quad (\text{A.4})$$

and

$$l^\pm = \frac{\gamma_v L}{2} \left(\frac{v}{u \cos \theta} \pm 1 \right) \quad (\text{A.5})$$

It follows from (A.4) and (A.5) that:

$$l^\pm = \gamma_v L^\pm \quad (\text{A.6})$$

In the limit $c \rightarrow \infty$, $\gamma_v \rightarrow 1$ of Galilean relativity, $l^\pm = L^\pm$, so the particle trajectories pass through the points A' and B' at the end of the train for any values, $\ll c$, of v and u . In this limit (A.2) becomes:

$$u^\pm \sin \theta^\pm = u \sin \theta \quad (\text{A.7})$$

showing that both particles have the same transverse velocity. They then arrive simultaneously at A' and B' in both the train and the embankment frames.

As is clear from inspection of Fig.9, this is no longer the case for photons, given by setting $u = c$ in (A.4) and (A.5). All the configurations in Fig.9 have $\theta = 60^\circ$. Fig.9a corresponds to $\cos \theta = \beta_v = 1/2$, so that $L^- = l^- = 0$ and a photon trajectory passes through B'. In general, however, (see Fig.9b) the photon trajectories do not pass through A' and B', so contrary to the naive (classical) expectation drawn from inspection of Fig.8b, detectors situated at A' and B' would register no photon detection events.

In Fig.9c, v is chosen so that the condition $L^+ = l^-$ is respected. In this case the photon trajectory at the larger angle relative to the x' -axis passes through A'. Setting $L^+ = l^-$, $u = c$ and $\cos \theta = 1/2$ in (A.4) and (A.5) gives a cubic equation for β_v :

$$4\beta_v^2(1 + \beta_v) + \beta_v - 8 = 0 \quad (\text{A.8})$$

The solution of this equation is $\beta_v = 0.950$ or $\gamma_v = 3.22$.

In Fig.10 are shown the configurations in the train frame (positions of the embankment and the photon bunches) for the three cases shown in Fig.9, when the photon bunches with largest angles to the x' -axis cross the latter. It can be seen that in no case, contrary to what is observed in the embankment frame, are the points A,A' or B,B' contiguous when the photon trajectories pass through A' or B'. If it is now imagined that photon detectors equipped with signal sources similar to Y', Y_U , Y_L and Y'' in Fig.4, register photon coincidences in the train for the configurations shown in Fig.10, the corresponding signals (calculated using the TD formula (2.11)), as observed in the embankment frame, are shown in Fig.11. Only for $\beta_v = \cos \theta = 1/2$ are the photon-B' coincidence in the train frame and the photon-B coincidence in the embankment frame observed simultaneously in the latter. This is easy to understand as the consequence, in this case, of exact compensation between the shorter path length in S' tending to make the photon-B' coincidence earlier than the photon-B one, and the TD effect that tends to make it later. Finally, in Fig.12 are shown the simultaneous photon-A and photon-B coincidence events of Fig.8b, as viewed from the train frame, for different values of β_v . In all cases these events are also simultaneous in the train frame. However, following Einstein's methodology in the original TETE and placing an observer at the middle of the train, this observer

sees always the photon-B coincidence events before the photon-A ones due to different light propagation time delays. The observer will then judge, by Einstein's criterion of simultaneity, that the events are not simultaneous, whereas, in fact, they are.

It is interesting to note the huge differences between the AA' and BB' separations on the embankment and in the train in Figs.10, 11 and 12, for corresponding event configurations in the train and on the embankment, when β_v is of order unity. This is the most deeply counter-intuitive effect among all the predictions for space-time experiments given by special relativity theory.

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